Analysis and design of Prestress concrete members to Eurocodes (EC2) considering the temperature

Anthony Rochefort Msc

Dissertation submitted in total fulfilment of the requirements of the degree of Msc Structural Engineer

School of Engineering and the Building Environment
Napier University

September 2008

Authorship declaration

I, Anthony Rochefort, confirm that this dissertation and the work presented in it are my own achievement.

1. Where I have consulted the published work of others this is always clearly attributed.
2. Where I have quoted from the work of others the source is always given. With the exception of such quotations this dissertation is entirely my own work.
3. I have acknowledged all main sources of help.
4. If my research follows on from previous work or is part of a larger collaborative research project I have made clear exactly what was done by others and what I have contributed myself.
5. I have read and understand the penalties associated with plagiarism.

Anthony Rochefort, 18/09/08

Matriculation number: 05010480

Acknowledgements

Two years ago I have been introduced to the prestressed concrete design which led to the choice of this dissertation. Credit for that teaching goes to Ben Zhang. I thank also Johnson Zhang, my tutor, for his advice. My gratitude goes to my partner, Amy Walters for help and patient and finally my parents, Gerard and Françoise Rochefort, which supported my journey to United Kingdom.

Abstract

Title: Analysis and design of Prestress concrete members to Eurocodes (EC2) considering the temperature effect

This dissertation examines the effects of the temperature on the prestress concrete because they are not well known about and can be a tragedy. The risks are cracking and failure of the structure. Inside this project a large part is devoted to the creep and the shrinkage effect and analysis of prestress loss because they influence significantly the design of prestressed concrete. The latest motive is to really undertake the losses in the prestressed concrete member. There are two major effects of applying the temperature profile on the prestress concrete member. The first is to increase the stress in the structure, and second is the loss of strength of the concrete and loss of prestress as the temperature rises, for example during a fire. However, the increase of stress in the members and the loss of strength in the concrete can be compensated by increasing the amount of prestressing steel, and by using higher class strength of concrete. The loss of prestress can be evaluated by calculation provided in the Eurocodes. For that reason the engineer's are currently trying to design with more accuracy, and of course using the Eurocodes will be the next step of the engineering design. The Eurocodes being the best efficient standard ever created. So, there is a need to design, analysis and understanding the prestressed concrete to Eurocode 2 considering the effects of temperature but also the effects of creep and shrinkage of concrete.

Contents

List of tables :

List of figures:

Symbols

A Accidental action

A Cross sectional area

A_c Cross sectional area of concrete

$A_{c,beam}$ Cross-sectional area of beam in composite construction

$A_{c,slab}$ Cross-sectional area of slab in composite construction

A_p Area of a prestressing tendon or tendons

A_s Cross sectional area of reinforcement

$A_{s,min}$ minimum cross sectional area of reinforcement

A_{sw} Cross sectional area of shear Reinforcement

b Overall width of a cross-section, or actual flange width in a T or L beam

b_v Length of side of critical perimeter

bw Width of the web on T, I or L beams

C Compression

d Diameter; Depth

e Eccentricity

D Diameter of mandrel

E Effect of action

$E_{c,eff}$ Effective modulus of elasticity of concrete

E_{cd} Design value of modulus of elasticity of concrete

E_{cm} Secant modulus of elasticity of concrete

E_{cmt} Lomg-term modulus of elasticity of Concrete

$E_{c,slab}$ Modulus of elasticity of slab concrete

E_p Design value of modulus of elasticity of prestressing steel

E_s Design value of modulus of elasticity of reinforcing steel

EQU Static equilibrium

F Action

F_k Characteristic value of an action

f_c Compressive strength of concrete

f_{cd} Design value of concrete compressive

$(f_{max})_{qp}$ Maximum allowable concrete stress under quasi-permanent load

$(f_{max})_{ra}$ Maximum allowable concrete stress under rare load

f'_{max} Maximum allowable concrete stress at transfer

f_{min} Minimum allowable concrete stress under rare load

f'_{min} Minimum allowable concrete stress at transfer

f_{yk} Characteristic yield strength of reinforcement

g Ratio of bond strength of prestressing and reinforcing steel

h Overall depth of a cross-section

i Radius of gyration

k Coefficient; Factor

l (or L) Length; Span

l0 effective length or lap length

m Mass

I Second moment of area of concrete section

L Length

m modular ratio E_s/E_{cm}

M Bending moment

Mmax Maximum bending moment

M_o Transfer bending moment

M_{qp} quasi-permanent bending moment

M_{ra} rare load bending moments.

N Axial force

P Prestressing force

P_e Effective prestress force after elastic shortening

P_o Initial force in tendons

p Loss of prestress force per unit length

Q_k Characteristic variable action

R Resistance

r Radius

r_{ps} Radius of curvature of tendons

strength	$1/r$ Curvature at a particular section
f_{ck} Characteristic compressive cylinder strength of concrete at 28 days	$1/r_b$ Curvature at midspan of beam or support of cantilever
f_{cm} Mean value of concrete cylinder compressive strength	$1/r_m$ Effective curvature
f_{ctk} Characteristic axial tensile strength of concrete	s_v Spacing of links
f_{ctm} Mean value of axial tensile strength of concrete	SLS Serviceability limit state
f_{cu} Characteristic compressive cube strength of concrete at 28 days	T Tension
f_p Tensile strength of prestressing steel	T_d Force in longitudinal reinforcement
f_{pk} Characteristic tensile strength of prestressing steel	T_f Final cable tension
$f_{p0,1\ 0,1\%}$ proof-stress of prestressing steel	T_o Initial cable tension
$f_{p0,1k}$ Characteristic 0,1% proof-stress of prestressing steel	u Perimeter of concrete cross-section, having area Ac
f_t Tensile strength of reinforcement	ULS Ultimate limit state
f_{tk} Characteristic tensile strength of reinforcement	V Shear force
w_d Dead load	V_{eff} Effective shear force in a slab
w_{fr} Frequent uniform load	V_{Rd1} Shear resistance of section without reinforcement
w_o Self weight	V_{Rd2} Maximum shear resistance of a section
w_{qp} Quasi-permanent uniform load	V_{sd} Ultimate shear force
w_{ra} Rare uniform load	w Uniformly distributed load
w_{ult} Ultimate uniform load	ε_{pk} Characteristic ultimate strain in tendons
x Neutral axis depth	ε_s Steel strain
x,y,z Coordinates	ε_{sh} Shrinkage strain
z Lever arm	ε_{sm} Average concrete strain at level of tendons
Z_b Elastic section modulus for the bottom fibre	ζ Distribution coefficient
$Z_{b,beam}$ Section modulus for bottom of beam in composite section	μ Coefficient of frictio
$Z_{b,comp}$ Section modulus for bottom of composite section	ϑ_w Minimum shear reinforcement ratio
Z_t Elastic section modulus for the top fibre	ϑ_1 Tension reinforcement ratio
$Z_{t,comp}$ Section modulus for top of composite section	$\Delta\sigma_p$ Reduction in tendon stress due to elastic shortening
α Short-term prestress loss factor	$\Delta\sigma_{pr}$ Variation of stress in the tendons due to steel relaxation
β Long-term prestress loss factor	σ Standard deviation
γ_f Partial factor of safety for load	σ_b Stress at bottom of section
	$\sigma_{b,beam}$ Stress at bottom of beam in composite section
	$\sigma_{b,slab}$ Stress at bottom of slab in composite section
	σ_{cg} Stress in concrete at level of tendons

γ_m Partial factor of safety for materials γ_p Partial factor of safety for prestressing δ Beam deflection δ_{ad} Anchorage draw-in δ_m Deflection of beam due to end-moments δ_o Deflection at transfer δ_{sh} Deflection due to shrinkage ε_c Concrete strain ε_{cu} Ultimate concrete strain ε_p Total strain in the tendons	σ_{cp} Axial stress in the concrete σ_{cpo} Initial stress in the concrete adjacent to the tendons due to prestress σ_t Stress at top of section $\sigma_{t,beam}$ Stress at top of beam in composite section $\sigma_{t,slab}$ Stress at top of slab in composite section T_{Rd} Basic concrete shear strength ϕ Creep coefficient

Introduction

Different approaches have been proposed to design the prestress concrete, from Freyssinet the creator of the prestress concrete in 1928, to the young generation of engineering. Nowadays in Europe the prestress concrete has to be designed with the latest code, the Eurocodes. The way to design and the philosophy of the design has not changed since the beginning of prestress concrete but the design has become more scientifically accurate, economically efficient and all effects are more and more understandable.

The purpose of this dissertation is to introduce the Eurocodes and to explain the principles of design for prestressed concrete structures and also to introduce the thermal, creep and shrinkage effect as it is explained in the Eurocodes and described by researchers in the past. Finally the dissertation gives some guidance of the design of prestress structure related to Eurocode 2. .

The context of the dissertation can be divided in several parts, the prestressed concrete introduction, the Eurocodes, the temperature, creep and shrinkage effects, the loss of prestress and finally some basic design. The issues of the dissertation is to find out how to understand different phenomenon happening on the prestress force and to analysis in the best way, Question such as "What is the effect of the temperature, creep and shrinkage in the prestress concrete?" will be answered. The study and observation of the topic have been revising and simplifying to ask the good question and find the good answer. For each of these matters, it is imperative to be critics on the past research for the most excellent realisation of the dissertation.

There are three main difficulty for this project. The first one is to read the Eurocodes and also surrounding text on this matter and to be able to identify the useful information. The difficulty is to be able to understand the peace of principles and rules. The second difficulty is to have a complete comprehension and knowledge of the prestress concrete to undertaken the subject. And finally the third difficulty is on the effect of temperature, creep and shrinkage to find data and make sure the value of this data are correct and to understand it. It is also part of physics matters less as structural engineering, than the improvement of the knowledge of the thermal actions and time-dependence effects are important.

The research on prestressed concrete has never stopped since the discovery of this technique in the 20^{th} century. Because of the difficulty and the wide surrounding information needs for understanding completely each part of the prestress concrete the researcher investigate step by step, every little part to improve, accurate and familiarise the knowledge of this technology. It still remains unclear the effects of the variation of the temperature, or the extreme temperature on the structures. Mechanism as joist has been used for building for the deformation but not real advancement in the design has been done. Research is difficult in this matter because you can not create a huge prestress beam and apply cold or/and hot in a laboratory. The problem as the same with the shrinkage and the creep phenomenon.

1- Basic Explanation

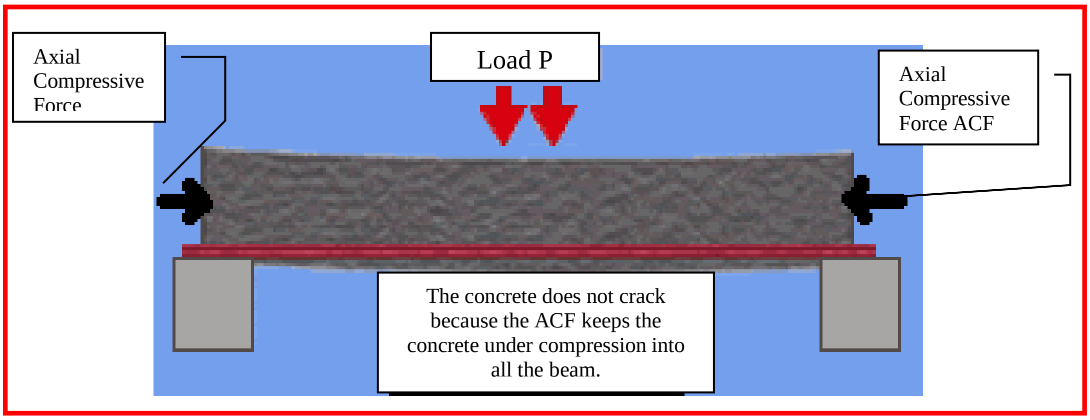
The prestressing concrete is today one of the most common ballast used for structure in building and civil engineering. Elegant and impressive structures have been built, from bridges across rivers to skyscrapers. The history (see annexe one for prestresssed concrete history) of prestressed concrete is rich with innovation and engineering brilliance. There are lists of benefits (see Table 1.1) to use prestressed concrete. The mechanism and the techniques for the prestressed running are particular so the prestressed concrete becomes unique technology.

Table 1.1: Benefits of prestressing

The benefits of prestressing on the structures
1. **The loads on the prestress structures may be more heavy as no prestress structure (i.e. Reinforced concrete members)** 2. **Tension and cracking under service loads may be eliminated or reduced, depending on the magnitude of the prestressing force.** 3. **Downward deflections of beams and slabs under service loads may be avoided or greatly reduced.** 4. **Fatigue resistance (i.e. the ability to resist the effect of repeated live loading due to, for instance, road and rail traffic) is considerably enhanced.** 5. **Segmental forms of construction, as in the row of books, become a practical reality.** 6. **Very high strength steel may be used to form the tendons (see figure 2)** 7. **Beam and slab sections may be smaller than in reinforced concrete, due mainly to capacity to reduce deflection.**

The concrete has strong resistance to compression but not to in tension. If we apply the axial compressive force (see Figure 1.1), the concrete is compressed everywhere and cancels the tension caused by the bending on the beam due to the load P. The concrete is then to be compressed under the neutral axis.

Figure 1.1: How the prestressing works



The main difference between prestressed or reinforced concrete, it is the steel. The prestressing steel is active and no passive. The steel used in the prestressed concrete is called tendon (strand, wire), (see Figure 1.6). The tendons apply loads (see Figure 1.3) to the concrete as a result of their prestress force, while in reinforced concrete the stresses in the reinforcement result from the loads applied to the structure. A proportion of the external loads is therefore resisted by applying a load in the opposite sense through the prestressing whilst the balance has to be resisted by ordinary reinforcement.

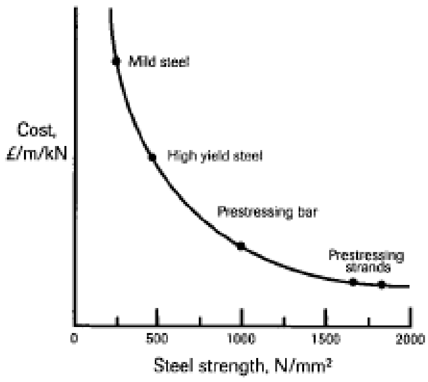


Figure 1.2:Relative unit cost (per unit force) of reinforcing and prestressing steel

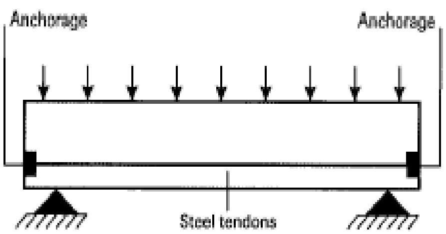


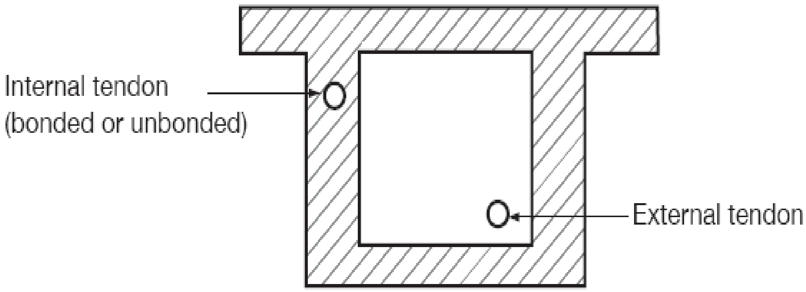
Figure 1.3: Prestressed concrete beam with steel tendons

<u>1.1- Prestressed tendons can be internal or external (see Figure 1.4):</u>

- Internal, i.e. within the concrete, either bonded to the concrete or unbonded (single strand in a plastic tube filled with grease).

- External, i.e. outside the concrete but inside the envelope of the member.

Figure 1.4 : Internal and external tendons



1.2- Pre-tensioning and post-tensioning prestress:

- The pretensioned members: it is when the tendons are stressed before the concrete is cast around them and the force transferred to the concrete when it has obtained sufficient strength.

- The post-tensioned members: it is when the ducts are cast into the concrete through which the tendons are threaded and then stressed after the concrete has gained sufficient strength.

There are different ways for implementation of pre-tensioning, post-tensioning. Both have advantages and disadvantages (see Table 1.2)

Pre-tensioning:

- The prestressing steels are tensioning on a casting bed.
- The concrete poured touches the steel wires.
- The concrete element is then put under stress by cutting the prestressed wires.
- During the curing stage the tendons bond to the surrounding concrete and compress the concrete.

This technique is often used for the building with precast members with the tendons bonded to the concrete. The precasting factories technique off site have permanent casting bed for reducing the cost of the pre-tensioning. Nevertheless, the pre-tensioned members are limited by transportation requirements and not used for construction like bridges.

Post-tensionning:

- The tendons are putted by jack at the extremity of the element on the concrete by anchorage after the concrete is poured and hardened.
- Often the tendons are not inside the concrete, which is external prestressing of course they are only for the extremity.

This technique is the common method of prestressing *in-situ members with* bonded or unbonded tendons because it does not require a casting bed. This is most often used for the viaducts, bridges, etc.

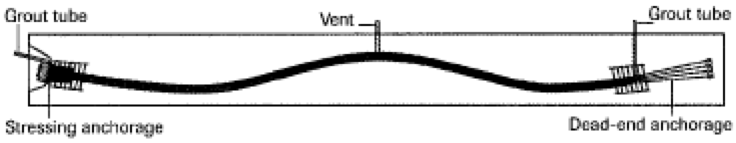
Table 1.2: Advantages and disadvantages of pre- and post-tensioning

Type of systems	Advantages	Disadvantages
Pretensioned	- no need for anchorages - tendons protected by concrete without the need for grouting or other protection - prestress is generally better distributed in transmission zones - factory produced precast units	- more difficult to incorporate deflected tendons - heavy stressing bed required
Post-tensioned	- no external stressing bed required - more flexibility in tendon layout and profile - draped tendons can be used - *in-situ* on site	- tendons require a protective system - large concentrated forces in end blocks

1.3- Bonded systems (see Figure 1.5):

Bonded tendons are installed in galvanised steel or plastic ducts that are cast into the concrete, which can be either circular or oval-shaped and change in size for the different number of steel strands within each duct. Metal ducts are made from either spirally wound or seam folded galvanised metal strip. The use of plastic ducts should be considered when designing car parks. The oval duct is used in conjunction with an anchorage, which ensures once the strands have been stressed the void around the strands is filled by cementations, which fully bonds the strands to the concrete. The strands are retained in the same plane in order to achieve maximum eccentricity.

Figure 1.5: Typical arrangement for internal post-tensioned tendons in bonded construction



1.4- Unbonded systems:

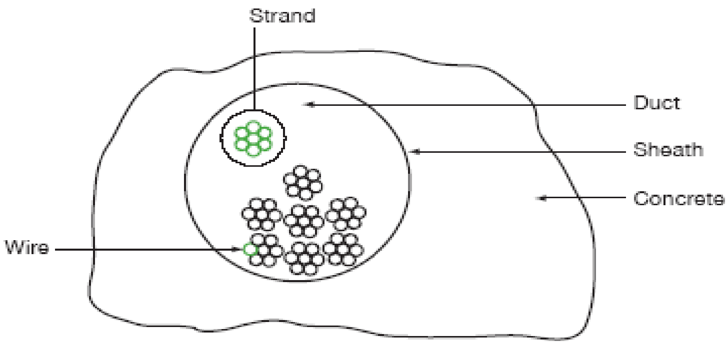
The individual steel strands are encapsulated in a PVC sheath and the voids between the sheath and the strand are filled with a rust-inhibiting grease that protect the unbonded tendons. The individual tendons are anchored at each end with anchorage castings. The sheath and grease are applied under factory conditions and the completed tendon is electronically tested to ensure that the process has been carried out successfully. The tendons are cast into the concrete section and are jacked to apply the required prestress force once the concrete has achieved the required strength.

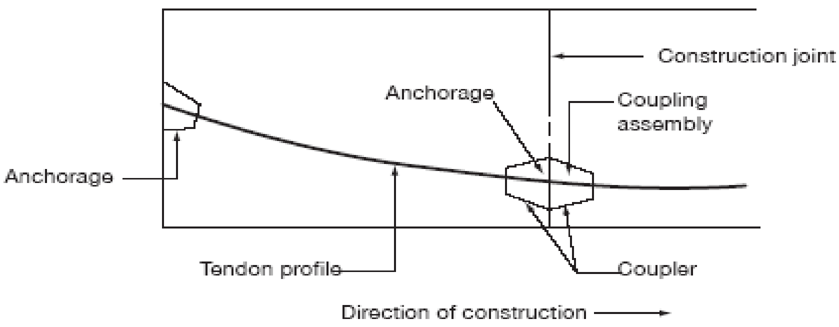
Table 1.3 lists the main characteristics of bonded and unbonded systems.

Table 1.3: Advantages and disadvantages of bonded and unbonded systems

Type of systems	Advantages	Disadvantages
Bonded	- Tendons are more effective at ULS (due to the strain compatibility with the concrete); - Does not depend on the anchorage after grouting; - Localises the effects of damage; - The prestressing tendons can contribute to the concrete shear capacity.	- Tendon cannot be inspected or replaced; - Tendon cannot be re-stressed once grouted; - Accidental damage to a tendon results in a local loss of the prestress force only; - Due to the concentrated arrangement of the strands within the ducts a high force can be applied to a small concrete section.
Unbonded	- Tendons can be removed for inspection and are replaceable if corroded; - Reduced friction losses; - Generally faster construction; - Tendons can be re-stressed; - Thinner webs and larger lever arm; - Tendons are flexible and can be easily fixed to different profiles (slab); - Tendon can be prefabricated off site.	- Less efficient at ULS; - Relies on the integrity of the anchorages and deviators; - A broken tendon causes prestress to be lost for the full length of that tendon; - Less efficient in controlling; cracking; - Careful attention is required in design to ensure against progressive collapse.

Figure 1.6: Nomenclature associated with prestressed concrete members





1.5- Structural behaviour:

Regard as a rectangular concrete member (see Figure 1.7) under load, prestress P and self weight (see Figure 1.8, 1.9 and 1.10). The whole section of the member is under stress, everywhere the section is $P/A_{c.}$ A_c is the area cross-section.

Figure 1.7: Rectangular concrete member

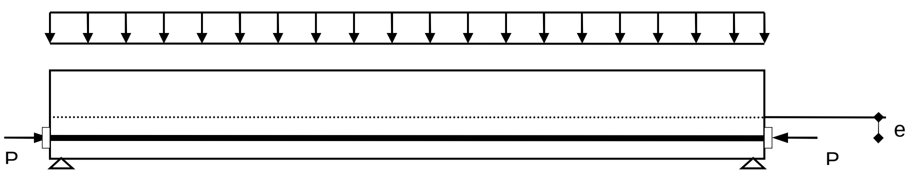


Figure 1.8: Eccentrically prestressed member

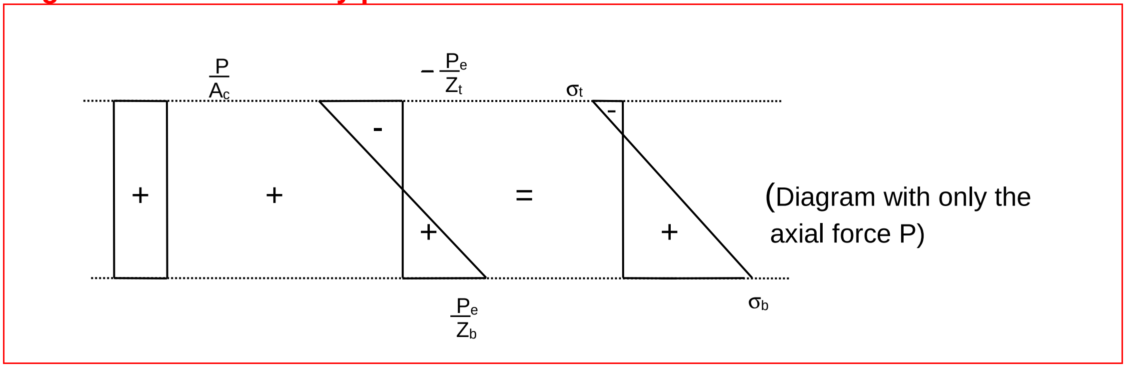


Figure 1.9: Stresses due to prestress and applied load

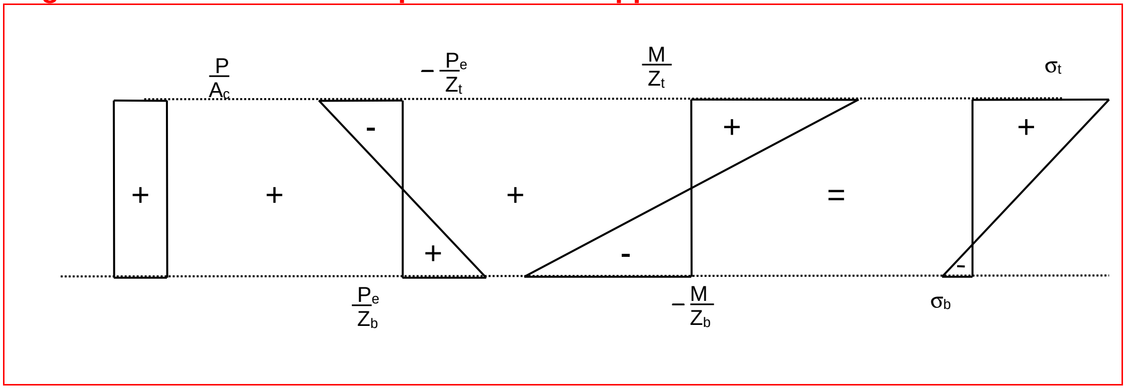
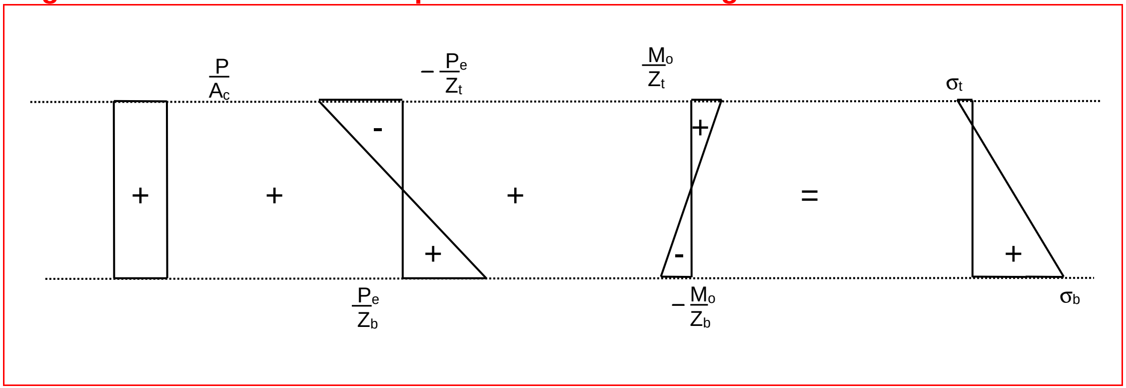


Figure 1.10: Stresses due to prestress and self weight



<u>1.6- Mechanism of prestressing force system (see Tables 1.4 and 1.5):</u>

In order to understand how the magnitude and position of the prestressing force affects the strength and stability of a beam, the mechanism of the system must be investigated. Table 1.4 illustrates a member under load and Table 1.5 illustrates the cracking.

Table 1.4: Mechanism of the systems, members under load

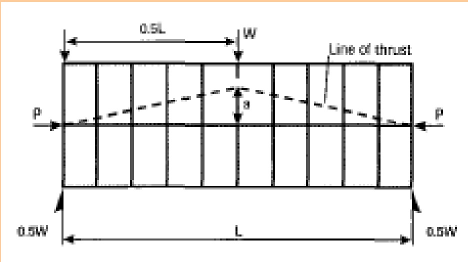
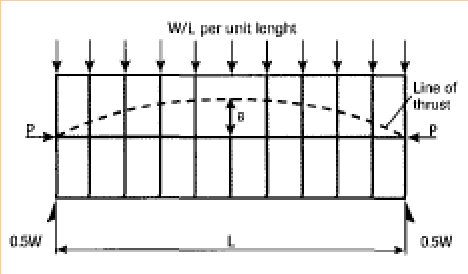
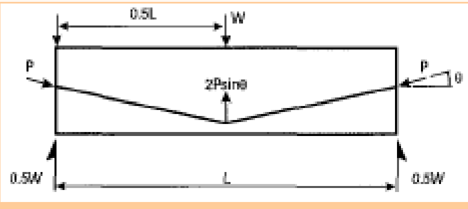
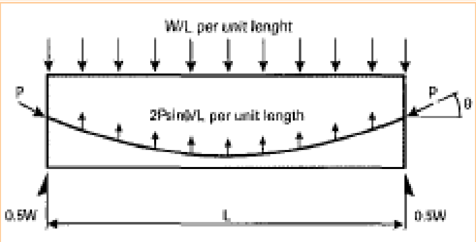
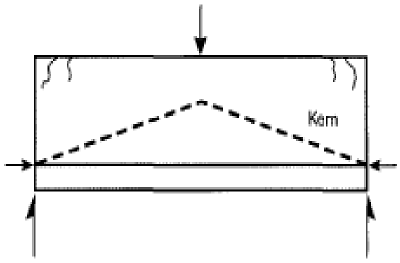
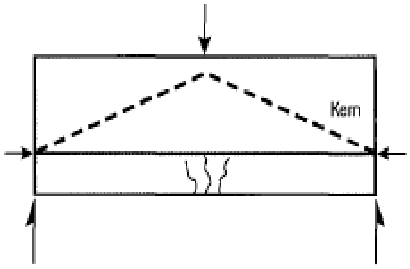
Load	Specification	Definition	Scheme
Prestressing unit with central point load	Consider the row of books	Pa= 0.5W * 0.5L W = 4Pa/L	
Prestressing unit with uniformly distributed load	Consider the row of books	Pa= 0.5W * (0.5L-0.25L) W = 8Pa/L	
Beam with central point load and deflected tendons	Deflected prestress	horizontal component prestress Q = Pcosθ	
Beam with uniformly distributed load and draped tendons	Draped prestress	horizontal component prestress Q = Pcosθ	

Table 1.5: Mechanism of the systems, members cracking

Craking	Definition	Scheme
Cracking at top of unit due to prestressing force being located below the kern	The load-carrying capacity of the unit and the location of the prestressing force are limited by the compressive stresses in the concrete and also by tension and cracking considerations. If tension is to be avoided entirely in a rectangular section, the force diagram should be kept within the middle-third of the section depth. This region, which varies with the shape of the cross-section, is called the kern of the section.	
Cracking at bottom of unit due to apex of force diagram rising above the kern		

1.7- The loss of prestress (see Figure 1.11):

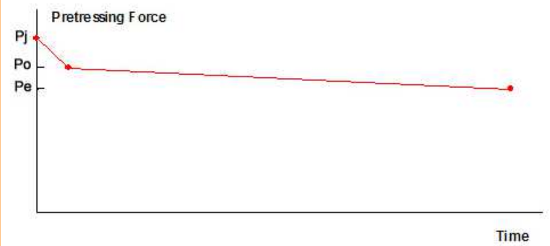
The prestressing force in the member progressively reduces with time. The loss of prestress has different causes. The loss can be immediate or short term and time related or long term. They need to be previewed and included in the calculation of pretressed concrete.

The sources of the loss of prestress at short-term are :

- elastic deformation of the concrete
- friction between tendon and tube
- tendon slip during anchoring.

The sources of the loss of prestress at long-term are:

- creep of the concrete
- shrinkage of the concrete
- relaxation of the steel.

Pj is the jacking force Po is the force immediately after transfer Pe is the final or effective force Pj-Po is the short-term loss Po-Pe is the long-term loss	 **Figure 1.11: Loss of prestress**

2- The Eurocodes

The Eurocode family started in 1975, There are ten Eurocodes (see Figure 2.1) covering all main structural materials produced by the

European Committee for Standardization (CEN) for replacing the national standards in 28 countries. I will be concentrate on Eurocode 0, Eurocode 1, Eurocode 2 and because these codes will be used for my design.

Figure 2.1: Eurocodes

BS EN 1990 (EUROCODE 0) Basis of structure design	Structural safety, serviceability and durability
BS EN 1991 (EUROCODE 1) Actions on structures	Actions on structures
BS EN 1992 (EUROCODE 2) Design of concrete structures	Structural dign and detailing
BS EN 1993 (EUROCODE 3) Design of steel structure	
BS EN 1994 (EUROCODE 4) Design of composite steel and concrete structures	
BS EN 1995 (EUROCODE 5) Design of timber structures	
BS EN 1996 (EUROCODE 6) Design of masonry structures	
BS EN 1999 (EUROCODE 9) Design of aluminium structures	
BS EN 1997 (EUROCODE 7) Geotechnical design	Geotechnical and seismic design
BS EN 1998 (EUROCODE 8) Design of structures for earthquake resistance	

Eurocode 0: Basis of structural design

The basis of structural design provides the information replicated in each of the material Eurocodes. It institutes principles and requirements for safety, serviceability and durability of the structures and also the best start for the approach to the Eurocodes to my project. I have scoped more important points for my project.

Every structure has some needs. For my project I will follow the main basic requirements explained in the Section 2 of the Eurocode 0.

<u>2-1 Basic requirements (from Section 2, clause 2.1):</u>

1) A structure shall be designed in such a way that it will, during its intended life, with appropriate degrees of reliability and in an economical way.

2) A structure shall be designed to have adequate

- structural resistance,
- serviceability,
- durability.

3) A structure shall be resistance to the fire for required period of time.

4) A structure shall be designed to any damage, e.g.

- Explosion,
- Impact,
- the consequences of human errors.

5) Potential damage shall be avoided or limited by appropriate choice of one or more of the following:

- avoiding, eliminating or reducing the hazards to which the structure can be subjected;
- selecting a structural form which has low sensitivity to the hazards considered;
- selecting a structural form and design that can survive adequately the accidental removal of an individual member or a limited part of the structure, or the occurrence of acceptable localised damage;
- avoiding as far as possible structural systems that can collapse without warning;
- tying the structural members together.

6) The basic requirements should be met:

- by the choice of suitable materials,
- by appropriate design and detailing,
- by specifying control procedures for design and use relevant to the partular project.

7) The provisions of Section 2 of Eurocode 2 should be interpreted on the basis that due skill and care appropriate to the circumstances is exercised in the design, based on such knowledge and good practice as is generally available at the time that the design of the structure is carried out.

2.2- Design working life (Section 2 2.3):

The design life for the structure should be specified (Table 2.1).

Table 2.1: Indicative design working Life (From NA to BS EN 1990:2002 – Table NA.2.1)

Design working life category	Indicative design working life (years)	Examples
1	10	Temporary structures
2	10 to 30	Replaceable structural parts, e.g. gantry girders
3	15 to 25	Agricultural and similar structures
4	50	Building structures and other common structures, not listed elsewhere in this table
5	120	Monumental building structures, highway, bridges, and other civil engineering structures

The documents How to design concrete structures using Eurocode 2, which summarise Section 6 of the Eurocode particularly the part 1 Introduction to Eurocodes have been used. Also, I used the information from the UK National Annex for Eurocode 0 NA to BS EN 1990:2002.

<u>2.3- Representative factors and the combinations value</u>:

Qk (Table 2.2) the characteristic value of single variable action is the principal representative value. All the other representative values are combination, which are obtained by applying the factors ψ_0, ψ_1, ψ_2 to the characteristic value. The factor ψ

differ with the type of imposed load and kind of construction (Example for Building Table 2.3).

Table 2.2: Representatives values

Representative values	Symbols	Definition
Characteristic value	Qk	Qk is determined statistically or by nominal value if there is insufficient value
Combination value	ψ_0 Qk	Its reduced probability of the simultaneous occurrence of two or more variable actions
Frequente Value	ψ_1 Qk	Used for a short time Used for the serviceability limit states (SLS) Used for the accidental ultimate limit state (ULS)
Quasi-Permanent value	ψ_2 Qk	Used for a considerable period of time Used for the long-term affects at the SLS and accidental and seismic ULS

Table 2.3: Value of y factors for buildings (from UK NA for Eurocode 0 - Table NA.A1.1)

Action	ψ_0	ψ_1	ψ_2
Imposed loads in buildings (see BS EN 1991-1-1)			
Category A: domestic, residential areas	0.7	0.5	0.3
Category B: office areas	0.7	0.5	0.3
Category C: congregation areas	0.7	0.7	0.6
Category D: shopping areas	0.7	0.7	0.6
Category E: storage areas	1.0	0.9	0.8
Category F traffic area, vehicle weight < 30 kn	0.7	0.7	0.6
Category G: traffic area, 30 kN < vehicle weight < 160 kN	0.7	0.5	0.3
Category H: roof (see BS EN 1991-1-1, Clause 3.3.2(1))	0.7	0	0
Snow loads on buildings (see BS EN 1991-3)			
For sites located at altitude H > 1000 m above sea level	0.7	0.5	0.2
For sites located at altitude H < 1000 m above sea level	0.5	0.2	0
Wind loads on buildings (see BS EN 1991-1-4)	0.5	0.2	0
Temperature (non-fire) in buildings (see BS EN 1991-1-5)	0.6	0.5	0

2.4- Combinations of actions:

The combinations of actions are specifically used for the definition of the magnitude of the actions to be applied when a limit state is under the influenced by different actions. The following process can be used to determine the value of actions used for analysis:

- Identify the design situation (e.g. persistent, transient, and accidental);
- Identify all realistic actions;
- Determine the partial factors for each applicable combination of actions;
- Arrange the actions to produce the most critical conditions.

For one variable action (e.g. imposed load) in a combination, the magnitude of the actions will be the multiplication between the correct partial factors and the value of the variable action.

For two or more variable actions in a combination, it needs to identify the leading action (Qk,1) and the other accompanying actions ($Q_{k,i}$). The accompanying action is always taken as the combination value.

Eurocode 2 indicates which combination should be used for which phenomenon.

There are two limit states: ULS and SLS.

2.5- Ultimate limit state (ULS) (Section 6 Clause 6.4):

The Eurocode lists four ultimate limit state to be considered in the design process:

EQU: Loss of equilibrium of the structure,
GEO: Failure or excessive deformation of the ground,
STR: Internal failure or excessive deformation of the structure or structural member,
FAT: Fatigue failure of the structure or structural members.

STR is the design process that will be used in my project. This is because I will calculate the deformation inside the structural prestressing member (e.g. beam).

The design values of actions (for the UK) used for persistent and transient design situations under the STR limit state, are listed in Table 2.4 and 2.5 as follows:

Table 2.4: Design values of actions, ultimate limit state – persistent and transient design situations (Table A1.2 (B) Eurocode)

Combination Expression reference	Permanent actions		Prestress (From table 2.4(B))	Leading variable action	Accompanying variable actions	
	Unfavourable	Favourable			Main (if any)	Others
Exp. (6.10)	$\gamma_{Gj,sup}G_{kj,sup}$	$\gamma_{Gj,inf}G_{kj,inf}$	$\gamma_P P$	$\gamma_{Q,1}Q_{k,1}$		$\gamma_{Q,i}\ \psi_{0,i}Q_{k,i}$
Exp. (6.10a)	$\gamma_{Gj,sup}G_{kj,sup}$	$\gamma_{Gj,inf}G_{kj,inf}$	$\gamma_P P$		$\gamma_{Q,1}\psi_{0,1}Q_{k,1}$	$\gamma_{Q,i}\ \psi_{0,i}Q_{k,i}$
Exp. (6.10b)	$\xi\gamma_{Gj,sup}G_{kj,sup}$	$\gamma_{Gj,inf}G_{kj,inf}$	$\gamma_P P$	$\gamma_{Q,1}Q_{k,1}$		$\gamma_{Q,i}\ \psi_{0,i}Q_{k,i}$
Note : Design for either Expression (6.10) or the less favourable of Expressions (6.10a) and (6.10b)						

Table 2.5: Design values of actions, derived for UK design, ultimate limit state – persistent and transient design situations

The expressions (6.10b) can be used when the permanent actions are not superior than 4.5 times the variables actions. (Except for storage loads (Category E, Table 3) where Expression (6.10a) always applies). For the typical concrete frame building, the expression (6.10b) will be the most structurally economical combination of actions.

For members supporting one variable action the combination is:

(0.925 * 1.35 Gk)+ 1.5 Qk = 1.2487 Gk + 1.5 Qk

Say : 1.25 GK + 1.5 Qk (Derived from Exp (6.10b))

2.6- Serviceability limit state (Section 6 Clause 6.5):

The eurocode list three combinations of actions at the serviceability state following table 2.6

Table 2.6: Combinations of actions

Combination	Permanent actions		Prestress	variable actions		Exemple to eurocode 2
	Unfavourable	Favourable		Main (if any)	Others	
Characteristic	$G_{kj,sup}$	$G_{kj,inf}$	P	$Q_{k,1}$	$\psi_{\iota,0}\Theta_{\iota,\kappa}$	
Frequent	$G_{kj,sup}$	$G_{kj,inf}$	P	$\psi_{\iota,1}\Theta_{1,\kappa}$	$\psi_{\iota,2}\Theta_{\iota,\kappa}$	Cracking-prestressing concrete
Quasi-permanent	$G_{kj,sup}$	$G_{kj,inf}$	P	$\psi_{2,i}Q_{k,1}$	$\psi_{\iota,2}\Theta_{\iota,\kappa}$	Deflection

2.7- Annex A1 and Annex A2:

Inside the Eurocode 0 there are two Annexes, the Annex A1 and Annex A2, applied for buildings and bridges, respectively.

Annex A1: Application for buildings

Field of application:

- Rules and methods for the combinations of actions,
- Design values of permanent, variable and accidental actions and ψ factors.

Annex A2: Application for bridges (National Annex for EN 1990 Annex A2)

Field of application:

- Rules and methods for the combinations of actions for serviceability and ultimate limit state verifications,
- Design values of permanent, variable and accidental actions and ψ factors for every kind of bridge,
- Rules and methods for the verifications relating to some material-independent serviceability limit states.

2.8- Relevant to prestress concrete inside Eurocode 0:

Characteristic values of actions (from Section 4, Clause 4.1.2)

- In cases where the structure is very sensitive to variations in G (e.g. some types of prestressed concrete structures), two values should be used even if the coefficient of variation is small. Then $G_{k,inf}$ is the 5% fractile and $G_{k,sup}$ is the 95% fractile of the statistical distribution for G, which may be assumed to be Gaussian.

- Prestressing (P) should be classified as a permanent action caused by either controlled forces and/or controlled deformations imposed on a structure. These types of prestress should be distinguished from each other as relevant (e.g. prestress by tendons, prestress by imposed deformation at supports).

NOTE The characteristic values of prestress, at a given time t, may be an upper value $P_{k,sup}(t)$ and a lower value $P_{k,inf}(t)$. For ultimate limit states, a mean value $P_m(t)$ can be used. Detailed information is given in EN 1992 to EN 1996 and EN 1999.

2.9- Design values of the effects of actions (from Section 6, Clause 6.3.2):

- In those cases where more refined methods are detailed in the relevant EN 1991 to EN 1999 (*e.g.* for prestressed structures) and reference to the next point.

- For non-linear analysis (i.e. when the relationship between actions and their effects is not linear), the following simplified rules may be considered in the case of a single predominant action :

a) When the action effect increases more than the action, the partial factor γ_F should be applied to the representative value of the action.

b) When the action effect increases less than the action, the partial factor γ_F should be applied to the action effect of the representative value of the action.

NOTE Except for rope, cable and membrane structures, most structures or structural elements are in category.

2.10- Combinations of actions (from Section 6, Clause 6.5.3) :

For the representative value of the prestressing action (i.e. P_k or P_m), reference should be made to the relevant design Eurocode for the type of prestress under consideration.

Eurocode 1: Actions on structures

Eurocode 1 contains ten parts (Table 2.7) giving details of a wide variety of actions. It is similar to the British Standards for the anticipated actions used in UK.

Table 2.7: Eurocode 1, its parts and dates publication

Reference	Title	Publication date	
		Eurocode	National Annex
BS EN 1991-1-1	Densities, self-weight and imposed loads	Apr-04	Dec-05
BS EN 1991-1-2	Actions on structures exposed to fire	Nov-04	
BS EN 1991-1-3	Snow loads	Jul-03	Dec-05
BS EN 1991-1-4	Wind actions	Apr-05	
BS EN 1991-1-5	Thermal actions	Mar-03	
BS EN 1991-1-6	Action during execution	Jul-05	
BS EN 1991-1-7	Accidental actions due to impact and explosions	Sep-06	
BS EN 1991-2	Traffic loads on bridges	Oct-03	
BS EN 1991-3	Actions induced by craned and machinery	Nov-06	
BS EN 1991-4	Actions in silos and tanks	Mar-06	

One of the noteworthy modification is the bulk density of reinforced concrete, which has been increased to 25 kN/m^3(Table 2.8). The National Annexes to Eurocode 1 give the imposed loads for UK.

Table 2.8: Selected bulk density of materials (from Eurocode 1, Part 1-1, Annex A)

Material	Bulk density (kN/m3)
Normal weight concrete	24.0
Reinforced or prestressed normal weight concrete	25.0
Wet normal weight reinforced concrete	26.0

2.11- Relevant to prestress concrete inside the Eurocode 1:

- ***The actions due to prestressing (from BS EN 1991-1-6 Section 4)***

a) Actions due to prestressing should be taken into account, including the effects of interactions between the structure and auxiliary construction works (e.g. falsework) where relevant and possible specific requirements defined for the individual project.

b) Loads on the structure from stressing jacks during the prestressing activities should be classified as variable actions for the design of the anchor region.

c) Prestressing forces during the execution stage should be taken into account as a permanent action.

Eurocode 2 contains four parts (Table 2.9). This new code has many benefits (Table 2.10), but there are some differences between Eurocode 2 and BS 8110 with which we need to be familiar. Eurocode 2 applies to design of buildings and civil engineering works. It conforms and combines with the other Eurocodes especially with EN 1990: Basis of structural design and EN 1991: Actions on structures, required for resistance, serviceability, durability and fire resistance of concrete structures.

Table 2.9: Eurocode 2, its parts and publication dates

Reference	Title	Publication date	
		Eurocode	National Annex
BS EN 1992-1-1	General rules and rules for buildings	Dec-04	Dec-05
BS EN 1992-1-2	General rules-Structural fire design	Feb-05	Dec-05
BS EN 1992-2	Concrete bridges - design and detailing rules	Dec-05	
BS EN 1992-3	Liquid-retaining and containment structures	Jul-06	

Part 1-1 of Eurocode 2 gives a general basis for the design of structures in plain, reinforced and prestressed concrete made with normal and light weight aggregates together with specific rules for buildings.

Part 2 of Eurocode 2 uses the general rules given in Part 1-1, (i.e. all the clauses for Basis of design and elements of materials and structural analysis etc …) to the design of concrete bridges. This part gives a basis for the design of bridges and parts of bridges in plain, reinforced and prestressed concrete made with normal and light weight aggregates.

Table 2.10: Benefits of using Eurocode 2

- **Learning to use the new Eurocodes will require time and effort on behalf**

of the designer, so what benefits will there be?

1. **The new Eurocodes are claimed to be the most technically advanced codes in the world.**
2. **Eurocode 2 should result in more economic structures than BS 8110.**
3. **The eurocodes are logical and organised to avoid repetition.**
4. **Eurocode 2 is less restrictive than existing codes.**
5. **Eurocode 2 is more extensive than existing codes.**
6. **Use of the Eurocodes will provide more opportunity for designers to work throughout Europe.**
7. **In Europe all public works must allow the Eurocodes to be used.**

Flow chart of process to prestressed concrete to the Eurocode 2

BS EN 1990 (EC0)
Basics of structure

BS EN 1991 (EC1)
Actions on structures

1991-1-1: General actions Densities, self weight, imposed loads for buildings
1991-1-2: General actions Actions on structures exposed to fire
1991-1-3: General actions Snow loads
1991-1-4: General actions Wind loads
1991-1-5: General actions Thermal actions
1991-1-6: General actions Construction loads
1991-1-7: General actions Accidental loads

BS EN 1997 (EC7)
Geotechnical design
BS EN 1998 (EC8)
Seismic design

BS EN 206-1
Concrete: Specification, performance, production and conformity BS 8500 Concrete. Complementary British Standard to BS EN 206-1

BS EN 1992 (EC2)
Design of concrete structures

1992-1-1 (EC2 Part 1-1)
General rules and rules for building
1992-1-2 (EC2 Part 1-2)
Structural fire design
1992-2 (EC2 Part 2)
Reinforced and prestressed concrete bridges
1992-3 (EC2 Part 3)
Liquid retaining and containing structures

BS EN 1994 (EC4)
Design of composite steel and concrete structures

1992-1-1: General rules and rules for building
1992-1-2: Structural fire design
1992-2 : Reinforced and prestressed concrete bridges

BS EN 10138:
Prestressing steels

Part 1: General requirements
Part 2: Wire
Part 3: Strand
Part 4: Bars

BS ENs for Precast Concrete Products

13369:* *Common rules for Precast Products
13225: Precast Concrete Products – Linear Structural Elements
14843: Precast Concrete Stairs
1168: Precast Concrete Products – Hollow core slabs
13224: Precast Concrete Products – Ribbed floor elements
13693: Precast Concrete Products – Special roof elements
14844: Precast Concrete - Box Culverts

BS EN 10080 :
Steel for the reinforcement of concrete
BS 4449 :
Steel for the reinforcement of concrete - Weldable reinforcing steel - Bar, coil and decoiled product
BS 4483 :
Steel fabric for the reinforcement of concrete
BS 8666 :
Specification for scheduling, dimensioning, bending and cutting of steel reinforcement for concrete

2.12- Familiarisation to Eurocode 2 (from How design concrete structures using Eurocode 2):

1) Eurocode 2 is generally laid out to give advice on the basis of phenomena (e.g. bending, shear, etc.) rather than by member types as in BS 8110 (e.g. beams, slabs, columns, etc.).

2) Design is based on characteristic cylinder strengths not cube strengths.

3) The Eurocode does not provide derived formulae (e.g. for bending, only the details of the stress block are expressed). This is the traditional European approach, where the application of a Eurocode is expected to be provided in a textbook or similar publication.

4) Units for stress are mega pascals, MPa (1 MPa = 1 N/mm^2).

5) Eurocode 2 uses a comma for a decimal point. It is expected that UK designers will continue to use a decimal point. Therefore to avoid confusion, the comma should not be used for separating multiples of a thousand.

6) One thousand is represented by ‰.

7) The partial factor for steel reinforcement is 1.15. However, the characteristic yield strength of steel that meets the requirements of BS 4449 will be 500 MPa; so overall the effect is negligible.

8) Eurocode 2 is applicable for ribbed reinforcement with characteristic yield strengths of 400 to 600 MPa. There is no guidance on plain bar or mild steel reinforcement in the

Eurocode, but guidance is given in the background paper to the UK National Annex10.

9) The effects of geometric imperfection (‘notional horizontal loads’) are considered in addition to lateral loads.

10) Minimum concrete cover is related to bond strength, durability and fire resistance. In addition to the minimum cover an allowance for deviations due to variations in execution (construction) should be included. Eurocode 2 recommends that, for concrete cast against formwork, this is taken as 10 mm, unless the construction is subject to a quality assurance system in which case it could be reduced to 5 mm or even 0 mm where non-conforming members are rejected (e.g. in a precast yard). It is recommended that the nominal cover is stated on the drawings and construction tolerances are given in the specification.

11) Higher strengths of concrete are covered by Eurocode 2, up to class C90/105. However, because the characteristics of higher strength concrete are different, some Expressions in the Eurocode are adjusted for classes above C50/60.

12) The ‘variable strut inclination’ method is used in Eurocode 2 for the assessment of the shear capacity of a section. In practice, design values for actual structures can be compared with tabulated values.

13) The punching shear checks are carried at 2*d* from the face of the column and for a rectangular column, the perimeter is rounded at the corners.

14) Serviceability checks can still be carried out using ‘deemed to satisfy’ span to effective depth rules similar to BS 8110. However, if a more detailed check is required, Eurocode 2 guidance varies from the rules in BS 8110 Part 2.

15) The rules for determining the anchorage and lap lengths are more complex than the simple tables in BS 8110. Eurocode 2 considers the effects of, amongst other things, the position of bars during concreting, the shape of the bar and cover.

The relevant to prestress concrete to Eurocode 2 are within design principle of prestress concrete to Eurocode 2.

3- Design of Prestress Concrete to Eurocode 2

3.1- Prestress (from EN 1992-1-1, Clause 2.3.1.4):

- The prestress is applied by tendons made of high-strength steel (wires, strands or bars).

- Tendons may be embedded in the concrete. They may be:

    - pre-tensioned and bonded,
    - post-tensioned and bonded or unbonded.

- Tendons may also be external to the structure with points of contact occurring at deviators and anchorages.

3.2- Design process and prerequisite relating to prestress:

The design process for the elements covers the design life, actions on structure, load arrangements, combinations of actions, analysis methods, materials properties, stability and imperfections, verifications. The particularities of prestressed concrete make it distinction with reinforced concrete. It explains inside clause 5.10 of Eurocode 2 Part 1-1.

1. The load arrangements: imposed snow and wind loads acting on a structure.

2. Combination of actions: the values of actions to be used when a limit state is under the influence of different actions.

3. Partials factors for prestress.

4. Materials properties: the properties of concrete and Prestressing steel (tendon, strand) and anchorage.

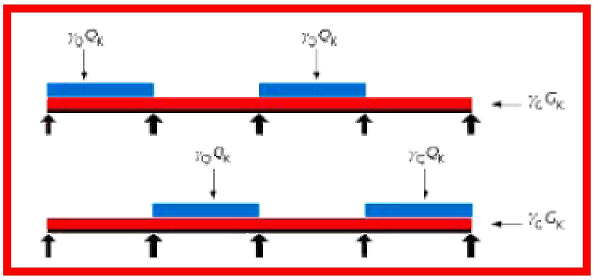
5. Structural Analysis: to find the distribution of internal forces and moments, losses of prestress, flexure, shear force, deflection.

3.3- The Load arrangements:

The variable actions are set up giving the most critical forces in structural members. There are three options decided by the UK for the national determination parameters (NDPs). (from national annex to Eurocode 2 Part1-1, Table NA.1)

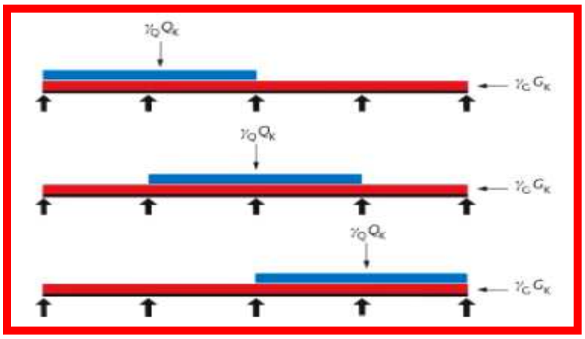
Option 1: Alternate or adjacent spans

Figure 3.1: Alternate spans loaded



a) Alternate spans carrying the design variable and permanent with other spans carrying only the design permanent load, (see Figure 3.1).

Figure 3.2: Adjacent spans loaded



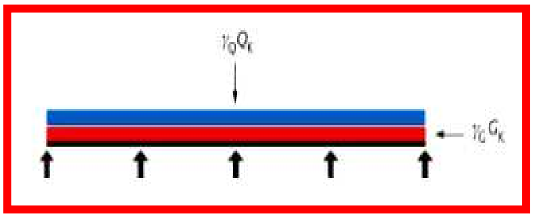
b) Any two adjacent spans carrying the design variable, and permanent all other spans carrying only the design permanent load, (see Figure 3.2).

Option 2: All or alternate spans

a) All spans carrying the design variable and permanent load, (see Figure 3.3).
b) Alternate spans carrying the design variable and permanent load, other spans carrying only the design permanent load, the same value γ_G should be used throughout the structure, (see Figure 3.1).

Option 3: For slabs, use the all spans loaded arrangement described option 2-a (see Figure 3.3)

Figure 3.3: All spans loaded



a) In a one-way spanning slab the area of each bay exceeds 30 m^2.
b) The ratio of the variable load Q_k to the permanent load G_k does not exceed 1,25.
c) The variable load Q_k does not exceed 5 kN/m2 excluding partitions.

3.4- Combination of actions (Clause 2.4.3):

The combinations of actions at ULS and SLS are given in EN 1990, Section 6.This means that the value of actions to be applied at a limit state is under the influence of different actions. The annex A1 for buildings and the annex A2 for bridges give the detail of expressions of the combination of actions. Explain part 'Eurocodes' of my report.

3.5- Partial factors for prestress (Clause 2.4.2.2):

The design value of prestress may be based on the mean value of the prestressing force given in EN 1990 Section 4.

Prestress in most situations is intended to be favourable and for the ultimate limit state verification the value of $\gamma_{P,fav}$ (Table 3.1) should be used. In the verification of the limit state for stability with external prestress, where an increase of the value of prestress can be unfavourable, $\gamma_{P,unfav}$ (Table 3.1) should be used. In the verification of local effects $\gamma_{P,unfav}$ (Table 3.1) should also be used.

Table 3.1: Value of partial factor for prestress (National Annex, Table NA.1)

Subclause	Nationally Determined Parameter	Eurocode recomandation	Uk decision
2.4.2.2 (1)	Partial factor for prestress $\gamma_{P,fav}$	1,0	0,9
2.4.2.2 (2)	Partial factor for prestress $\gamma_{P,unfav}$	1,3	1,1
2.4.2.2 (3)	Partial factor for prestress $\gamma_{P,unfav}$ for local effects	1,2	Use the recommended value

3.6- Material properties:

3.6.1- Concrete:

The properties of concrete is specified in conformity with BS EN 206-1 and BS 5328.The selection of appropriate and specification of concrete type and strength (Table 3.2) will be influenced by durability requirements, resistance requirements, material availability and basic economics. The strength of the concrete and the deformations during every steps of the transfer of prestress are the main factors important to prestressing.

The strength of concrete when the transfer of prestress is applied must be adequate, this means a concrete with a high early strength. The strength of hardened concrete, which increases with age should be taken at 28 days, for example:

- With the pre-tensioning:

  - For flooring units, the strengths are typically 28 - 40 N/mm² at transfer and 50 - 60 N/mm at 28 days.
  - For standard bridge beams, the values are typically 40 N/mm² at transfer and 60 N/mm² at 28 days.

- With post-tensioning, the age at transfer is less critical and accelerated curing is normally not necessary.

  - For floors in buildings, the strengths are typically 25 N/mm² at transfer and 40 N/mm² at 28 days.
  - The strength at 28 days is typically 50 N/mm² for bridges.

Table 3.2(a) : Strength and elastic modulus concrete

Strenght classes for concrete										Analytical relation/ explanation
f_{ck} (Mpa)	12	16	20	25	30	35	40	45	50	
$f_{ck,cube}$ (MPa)	15	20	25	30	37	45	50	55	60	
f_{cm} (MPa)	20	24	28	33	38	43	48	53	58	$f_{cm} = f_{ck} + 8$ (Mpa)
f_{ctm} (MPa)	1.6	1.9	2.2	2.6	2.9	3.2	3.5	3.8	4.1	$f_{ctm} = 0.30 f_{ck}^{(2/3)}$ < C50/60
$f_{ctk,0.05}$ (MPa)	1.1	1.3	1.5	1.8	2.0	2.2	2.5	2.7	2.9	$f_{ctk\ 0.05} = 0.7 f_{ctm}$ 5% fractile
$f_{ctk,0.95}$ (MPa)	2.0	2.5	2.9	3.3	3.8	4.2	4.6	4.9	5.3	$f_{ctk\ 0.95} = 1.3 f_{ctm}$ 95% fractile
E_{cm} (GPa)	27	29	30	31	33	34	35	36	37	$E_{cm} = 22\,(f_{cm}/10)^{0.3}$ (f_{cm} in MPA)

Table 3.2(b): Allowable concrete compressive stresses

Compressive stress at transfer	Compressive stress at service	
	Rare load combination	Quasi permanent loads
0.6 f'ck (N/mm²)	0.6 fck (N/mm²)	0.45 (N/mm²)

Lightweight concrete :

This kind of concrete is often used for the prestressed concrete structures because of the diverse performances (see Table 3.3).

Table 3.3 : Performance of lightweight concrete

Characteristics	Advantages
density is equal to 1300-2000kg/m^3	saving in Weight
strength dependent of the strength of the aggregates at 28 days:1N/mm² using a clay; to 50N/mm² using a pulverized-fuel-ash.	better fire resistance Insulation properties
with no information from the aggregate supplier the shrinkage = 400-600 * 10^{-6} and the specific creep = 0.7-0.9 * 10^{-4}	Shrinkage and creep are usually greater than standard concrete

3.6.2- Creep of concrete (Clause 3.1.4):

Most materials are affected by the creep which is a time-dependent deformation under constant load. The creep of concrete influences the long-term deflections and the loss of prestress force in prestressed concrete members. The creep slowly losses moisture in the concrete, affecting contraction in the structure of the cement paste in the concrete. The creep is more prominent in the prestressed concrete than reinforced concrete because the whole section is under compression.
A useful parameter to indicate the creep is the specific creep, this is the creep strain per unit stress. The long-term (70-year) specific creep strain may be established from the relationship:

$$\text{Specific creep} = \phi / E_{cmt}$$

Note: E_{cmt} is the elasticity modulus of the concrete in the long-term. ϕ is a creep coefficient (it can be obtained from table 3.4, when no more specific data are available). Ac and u are respectively the cross section area and perimeter.

Table 3.4 : Creep coefficients ϕ to Eurocode 2

Age at loads t_0 (Days)	Notional size h_0 (=2Ac/u) (mm)					
	50	150	600	50	150	600
	Dry atmospheric conditions (inside) (relative humidity 50%)			Humid atmospheric conditions (outside) (relative humidity 80%)		
1	5.5	4.6	3.7	3.6	3.2	2.9
7	3.9	3.1	2.6	2.6	2.3	2.0
28	3.0	2.5	2.0	1.9	1.7	1.5
90	2.4	2.0	1.6	1.5	1.4	1.2
365	1.8	1.5	1.2	1.1	1.0	1.0

Sometimes for the calculation of superstructure construct by phases, the creep calculation take into account:

- a non proportional load response
- creep speed depending:
  - concrete composition
  - surrounding hygrometry
  - size of the member
  - degrees of steel
- the load history

3.6.3- Shrinkage of concrete (Clause 3.1.4):

The excess of water in the concrete that has not been used to hydrate the cement evaporates. This is called the shrinks of the concrete member. The quantity of shrinkage varies on the environmental conditions surrounding the concrete but the external load on the member does not affect the shrinkage. Shrinkage of concrete changes with time. In the absence of more detailed data, the long-term shrinkage strain for normal concrete mixes to be used for design purposes may be funded from Table 3.5

Table 3.5.1 : Eurocode 2 shrinkage strains

Location of the member	Relative humidity %	Notional size h_0 (mm)	
		<150	600
Inside	50	600×10^{-6}	500×10^{-6}
Outside	80	330×10^{-6}	280×10^{-6}

It distinguishes the volume contraction by drying shrinkage ε_{cd} from the evaporation of water contain in the concrete and the endogen shrinkage ε_{ca}. The value of ε_{cd} is function of the followings parameters:

- water degree in concrete
- Cement dosage and usually the degree of ultra fine
- Surrounding drying
- Thickness of the structural member

The drying shrinkage has generally for value the data in table 3.52

Table 3.5.2: Deformation by drying shrinkage

Relative humidity %	Concrete strength (Mpa)	
	40	80
40	0.48 ‰	0.31 ‰
80	0.25 ‰	0.16 ‰

The endogen shrinkage ε_{ca} (in ‰) is evaluated in infinite time by this formula:

$$\varepsilon_{ca,\infty} = 2.5 \times (f_{ck}-10) \times 10^{-4}$$

Creep and shrinkage strains depend on the type of concrete and its environment. The effects of creep and shrinkage on loss of prestress force will be discussed in loss of prestress chapter.

3.6.4- Fire resistance:

Prestressed concrete works as fire resistant because it is ruled by the loss of strength of the steel with increase in the temperature rather than loss at the concrete strength. The Part 10 of Eurocode 2 discusses of the fire resistance requirements. The nominal cover depends of the periods of fire resistance and type of structural elements.

3.6.5- Prestressing steel, tendons and strands (Clause 3.3):

The properties of prestressing steel are specified in EN 10138, Parts 2 to 4 or European Technical Approval. This paragraph applies to wires, bars and strands are used like prestressing tendons in concrete structures.

The prestressing tendons are classified according to:

- Strength, denoting the value of the 0,1% proof stress ($f_{p0,1k}$) and the value of the ratio of tensile strength to proof strength ($f_{pk}/f_{p0,1k}$) and elongation at maximum load (ε_{uk})
- Class of relaxation,
- Size,
- Surface characteristics.

3.6.6- Relaxation of steel:

The phenomenon is similar to the creep of concrete in that it is time-dependent deformation under constant load. The quantity of relaxation depends on time, temperature and level of stress.

There are three classes of relaxation in Eurocode 2 (see Table 3.6). The design calculations for the losses due to relaxation of the prestressing steel should be based on the value of ρ_{1000}. The relaxation loss (in %) at 1000 hours after tensioning and at a mean temperature of 20 °C (see Figure 3.4). The long term values of the relaxation losses may be estimated for a time *t* equal to 500 000 hours (i.e. around 57 years).

Table 3.6: Classes of relaxtion

Class	Definition	P1000 value	Expression
Class 1	wire or strand - ordinary relaxation	8%	(3.28) $\Delta\sigma_{pr}/\sigma_{pi} = 5{,}39\ \rho_{1000}\ e^{6{,}7\mu}\,(t/1000)^{0.75(1-\mu)}\ 10^{-5}$
Class 2	wire or strand - low relaxation	2.5%	(3.29) $\Delta\sigma_{pr}/\sigma_{pi} = 0{,}66\ \rho_{1000}\ e^{9{,}1\mu}\,(t/1000)^{0.75(1-\mu)}\ 10^{-5}$
Class 3	hot rolled and processed bars	4%	(3.30) $\Delta\sigma_{pr}/\sigma_{pi} = 1{,}98\ \rho_{1000}\ e^{8\mu}\,(t/1000)^{0.75(1-\mu)}\ 10^{-5}$

Note:

$\Delta\sigma_{pr}$ is absolute value of the relaxation losses of the prestress

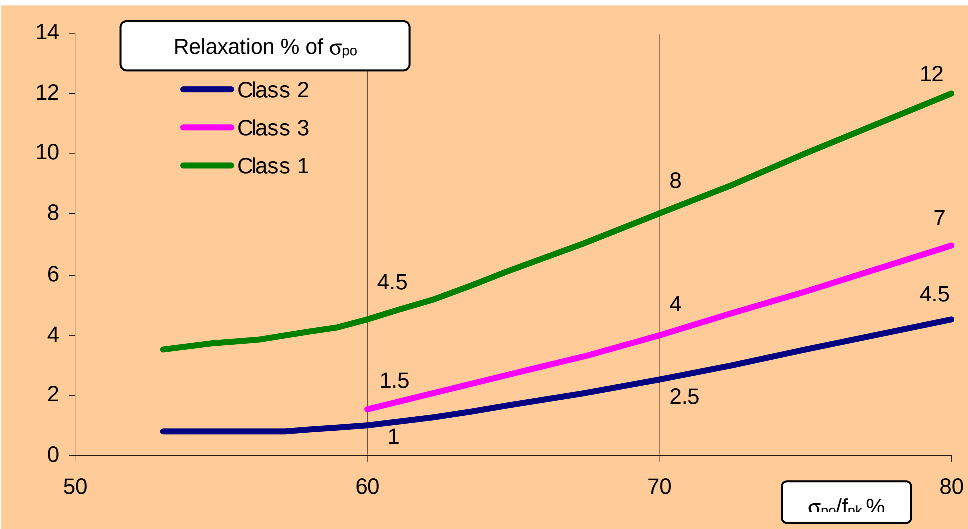
σ_{pi} For post-tensioning σ_{pi} is the absolute value of the initial prestress $\sigma_{pi} = \sigma_{pm0}$

For pre-tensioning σ_{pi} is the maximum tensile stress applied to the tendon minus the immediate losses occurred during the stressing process

t is the time after tensioning (in hours)

$\mu = \sigma_{pi}/f_{pk}$, where f_{pk} is the characteristic value of the tensile strength of the prestressing steel

Figure 3.4: Relaxation of steel at 20°C after 1000h



Actually, the stress loss from relaxation in function of time t is illustrate in hour as:

$$\Delta\sigma pr = \sigma pi \times k1 \times \rho 1000 \times e^{(k2\times\mu)} \times (\frac{t}{1000})^{0.75\times(1-\mu)} \times 10^{-3}$$

The parameters k1 and k2 depend of the steel class. It is usually the manufacturer which gives the relaxation rate corresponding to the steel.

Table 3.7: classes of relaxation and K1 and K2 values.

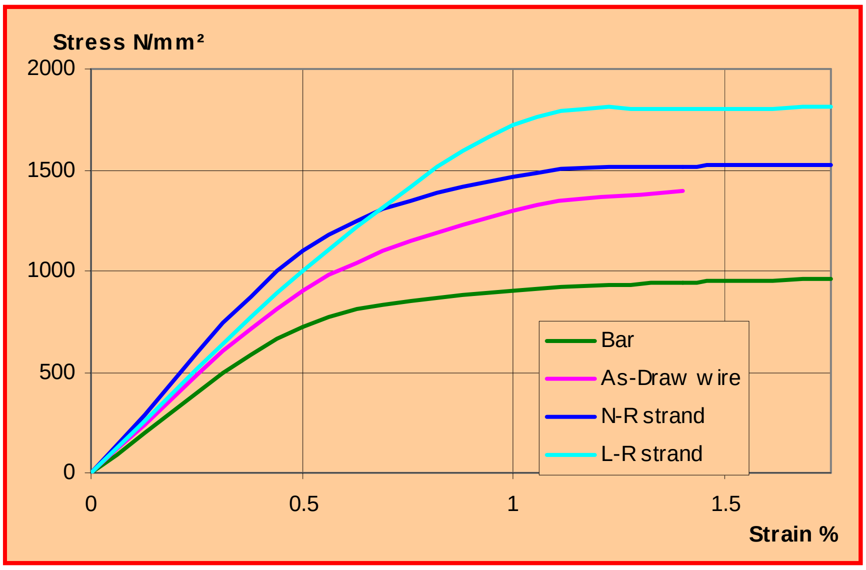
Class	Definition	P1000 value	K1	K2
Class 1	wire or strand - ordinary relaxation	8%	5.39	6.7
Class 2	wire or strand - low relaxation	2.5%	0.66	9.1
Class 3	hot rolled and processed bars	4%	1.98	8

For important structure it is usually and quite exclusively use class 2 because the strand have low relaxation.

3.6.7- Stress-strain curves for prestressing steel:

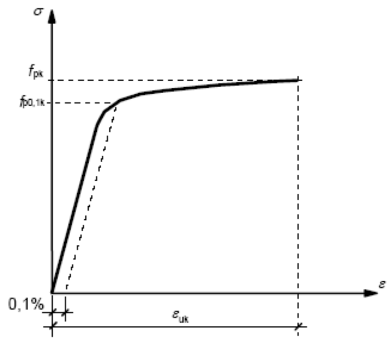
The figure 3.5 shows the stress-strain curves for prestressing steel. There is a difference between the mild steel and the high-strength steel. The high-strength steel does not possess the same well-defined yield point and also changed the proof stress. The 0,1% proof stress ($f_{p0,1k}$) and the specified value of the tensile strength (f_{pk}) are defined as the characteristic value of the 0,1% proof load and the characteristic maximum load in axial tension respectively, divided by the nominal cross sectional area as shown in figure 3.6.

Figure 3.5: Stress-strain curves for prestressing steel



The heat-treating process applied to the as-draw wire does not only reduce the relaxation of the steel, but also augment the proof stress and consequently extending the linear elastic range. The Eurocode 2 for the design purposes states the modulus of elasticity equal at 200 × 10^3 N/mm² for all types of steel.

Figure 3.6: Stress-strain diagram for typical prestressing steel



3.6.2.3- Corrosion of steel:

Prestressing tendons in sheaths (e.g. bonded tendons in ducts, unbonded tendons etc.) needs to be adequately and permanently protected against corrosion.(from clauses 3.3.7)

The prestressing steel has to be protected from the attack by moisture permeating the surrounding concrete.

- Pre-tensioned tendons must have the adequate cover and a concrete with a sufficiently low water/cement ratio. The main danger is at the end of the elements.

- Post-tensioned members have corrosion of partly-grouted tendons, thus this results in a failure to the ducts, all post-tensionned tendons ought to be left unbonded. A mix of greasing and coasting with plastic has been used successfully as an alternative protection to the tendons. The inspection of the tendons (i.e for the bridges) is made during the design life.

There is also the stress corrosion affecting wires and strands, this is from a breakdown of the structure of the steel itself. Small cracks emerge and the steel becomes fragile.

UK NA to EC2 Part1-1, Table NA.2 shows the cover to protect the corrosion of steel according to the normal weight quality of concrete, diverses classes of exposure and times.

<u>3.6.9- Ductility characteristics (from Clause 3.3.4) and fatigue (Clause 3.3.5):</u>

- The prestressing tendons shall have adequate ductility in elongation and bending, as specified in EN 10138 and the adequate ductility in tension may be assumed for the prestressing tendons if $f_{pk}/f_{p0,1k} \geq k$. stress-strain diagrams for the prestressing tendons, based on production data, shall be prepared and made available by the producer.

- Prestressing tendons shall have adequate fatigue strength.

If the steel is subject to a lot of stresses variations (between σmin and σmax), it is demonstrate after a high number of cycles, a steel rupture without the failure stress being rise. This phenomenon called fatigue. It is quite unknown in the theory but it is possible to determine the rupture parameters by experimentation.

The average number of cycles necessary to obtain the rupture will be smaller as the stress variation is bigger and the average stress is high.

Woehler studied the prestressing steel under fatigue effect. He takes a very high number of sample and he takes the steel tensioned to 0.75 time to its elastic limit.

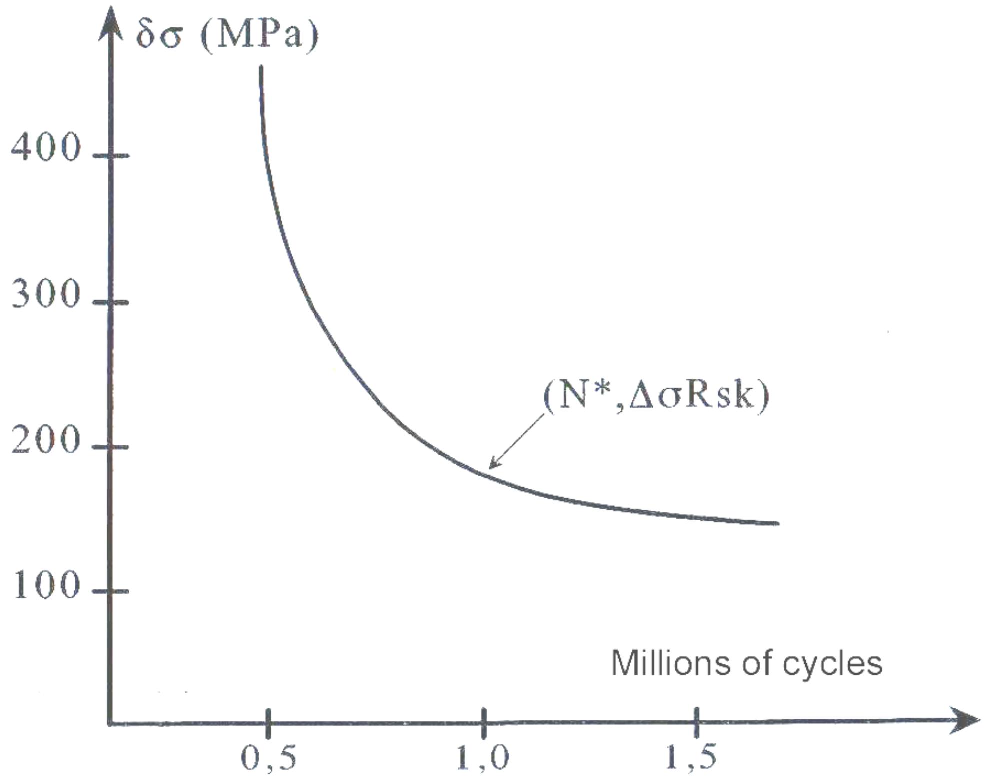


Figure 3.7: Resistance of the prestressing steel under the fatigue effect

The risk of fatigue is practically inexistent durin $\delta\sigma p < 100$ Mpa which is when the prestressed concrete without crack.

The Eurocode (6.8.4) present the model of the damage S-N curve by fatigue with exponentially form.

$N < N^*$ $(\Delta\sigma)^{k1} . N = \text{constant}$

$N > N^*$ $(\Delta\sigma)^{k2} . N = \text{constant}$

The numerical parameters are giving in the following table

Table 3.8 S-N Curves parameters

S-N curve of prestressing steel used for		stress exponent		
Pre-tensioning	N*	k1	k2	at N* cycles
Post-tensioning	106	5	9	185
- single strands in plastic ducts	106	5	9	185
- straight tendons or curved tendons in plastic ducts	106	5	10	150
- curved tendons in steel ducts	106	5	7	120

<u>3.6.10- Design assumptions (Clause 3.3.6):</u>

The Eurocode states seven points to follow for the design which are:

1) Structural analysis is performed on the basis of the nominal cross-section area of the prestressing steel and the characteristic values $f_{p0,1k}$, f_{pk} and ε_{uk}.

2) The design value for the modulus of elasticity, E_p may be assumed equal to 205 GPa for wires and bars. The actual value can range from 195 to 210 GPa, depending on the manufacturing process. Certificates accompanying the consignment should give the appropriate value.

3) The design value for the modulus of elasticity, E_p may be assumed equal to 195 GPa for strand. The actual value can range from 185 GPa to 205 GPa, depending on the manufacturing process. Certificates accompanying the consignment should give the appropriate value.

4) The mean density of prestressing tendons for the purposes of design may normally be taken as 7850 kg/m^3.

5) The values given above may be assumed to be valid within a temperature range between -40°C and +100°C for the prestressing steel in the finished structure.

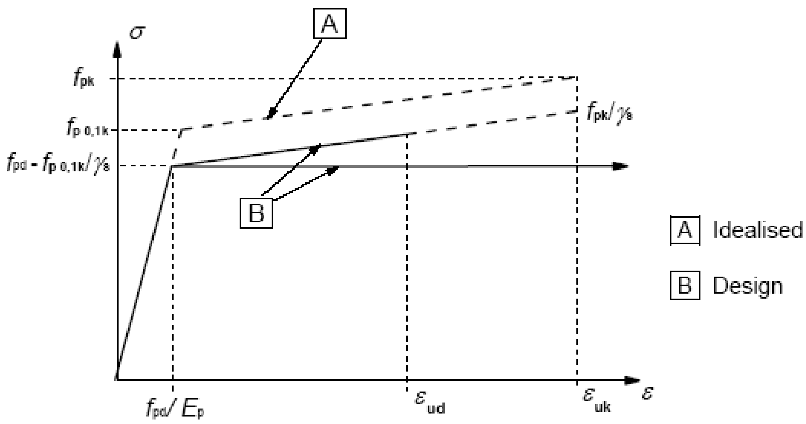
6) The design value for the steel stress, f_{pd}, is taken as $f_{p0,1k}/\gamma_S$ (see Figure 3.7).

7) For cross-section design, either of the following assumptions may be made (see Figure 3.7):

- an inclined branch, with a strain limit ε_{ud}. The design may also be based on the actual stress/strain relationship, if this is known, with stress above the elastic limit reduced analogously with Figure 3.7
- a horizontal top branch without strain limit.

The value of ε_{ud} for the UK may be found inside the National Annex table N.A.1, Subclause 3.3.6 (7). The recommended value is $0{,}9\ \varepsilon_{uk}$. If more accurate values are not known the recommended values are $\varepsilon_{ud} = 0{,}02$ and $f_{p0,1k}/f_{pk} = 0{,}9$.

Figure 3.7: Idealised and design stress-strain diagrams for prestressing steel (absolute values are shown for tensile stress and strain)



3.6.11- Prestressing tendons in sheaths (Clause 3.3.7):

Prestressing tendons in sheaths shall be adequately protected against the effects of fire explain inside EN 1992-1-2.

3.6.12- Prestressing General devices (Clause 3.4.1):

Anchorages and couplers apply to the anchoring devices and coupling devices for the application in post-tensioned construction, where:

- anchorages are used to transmit the forces in tendons to the concrete in the anchorage zone
- couplers are used to connect individual lengths of tendon to make continuous tendons.

Anchorages and couplers for the prestressing system considered shall be in accordance with the relevant European Technical Approval and the anchorages zones shall be with 5.10, 8.10.3 and 8.10.4.

3.7- Mechanical properties (Clause 3.4.1.2):

3.7.1- Anchored tendons:

Prestressing tendon anchorage assemblies and prestressing tendon coupler assemblies shall have strength, elongation and fatigue characteristics sufficient to meet the requirements of the design and in accordance with the appropriate European Technical Approval.

3.8- External non-bonded tendons, the device in Eurocode 2 (Clause 3.4.2):

An external non-bonded tendon is a tendon situated outside the original concrete section and is connected to the structure by anchorages and deviators only. The post-tensioning system for the use with external tendons shall be in accordance with the appropriate European Technical Approval. The minimum radius of curvature of the tendon in the anchorage zone for non-bonded tendons should be given in the appropriate European Technical Approval. For prestressing tendons, the minimum cover of the anchorage should be in accordance with the appropriate European Technical Approval. The minimum cover values for prestressing tendons in normal weight concrete taking account of the exposure classes and the structural classes is given by $c_{min,dur}$.

Note: Structural classification and values of $c_{min,dur}$ for use in a Country may be found in its National Annex. The recommended Structural Class (design working life of 50 years) is S4 for the indicative concrete strengths and the recommended modifications to the structural class is given in table 3.7 (from Table 4.3N to Eurocode 2). The recommended minimum Structural Class is S1. The recommended values of $c_{min,dur}$ for prestressing steel are given in table 3.10 (From Table 4.5N to Eurocode 2)

Table 3.9: Structural classification

Structural Class							
Criterion	Exposure Class according to table 3.11						
	X0	XC1	XC2/XC3	XC4	XD1	XD2/XS1	XD3/XS2/XS3
Strength Class (reduce class by 1)	≥ C30/37	≥ C30/37	≥ C35/45	≥ C40/50	≥ C40/50	≥ C40/50	≥ C45/55

Table 3.10: Values of minimum cover, $c_{min,dur}$ requirements with regard to durability for prestressing steel

Environmental Requirement for $c_{min,dur}$ (mm)							
Structural Class	Exposure Class according to table 3.11						
	X0	XC1	XC2/XC3	XC4	XD1/XS1	XD2/XS2	XD3/XS3
S1	10	15	20	25	30	35	40
S2	10	15	25	30	35	40	45
S3	10	20	30	35	40	45	50
S4	10	25	35	40	45	50	55
S5	15	30	40	45	50	55	60
S6	20	35	45	50	55	60	65

UK National Annex part 161 give c_{min} dependent the cement/combination types in the table NA.2.

Table 3.11: Exposure classes related to environmental conditions in accordance with EN 206-1

Class designation	Description of the environment	Informative examples where exposure classes may occur
1 No risk of corrosion or attack		
X0	For concrete without reinforcement or embedded metal: all exposures except where there is freeze/thaw, abrasion or chemical attack. For concrete with reinforcement or embedded metal: very dry	Concrete inside buildings with very low air humidity
2 Corrosion induced by carbonation		
XC1	Dry or permanently wet	Concrete inside buildings with very low air humidity Concrete permanently submerged in water
XC2	Wet, rarely dry	Concrete surfaces subject to long-term water contact Many foundations
XC3	Moderate humidity	Concrete inside buildings with moderate or high air humidity External concrete sheltered from rain
XC4	Cyclic wet and dry	Concrete surfaces subject to water contact, not within exposure classXC2
3 Corrosion induced by chlorides from sea water		
XD1	Moderate humidity	Concrete surfaces exposed to airborne chlorides
XD2	Wet, rarely dry	Swimming pools Concrete components exposed to spray industrial waters containing chlorides
XD3	Cyclic wet and dry	Parts of bridges exposed to spray containing chlorides Pavements Car park slabs
4 Corrosion induced by chlorides from sea water		
XS1	Exposed to airborne salt but not in direct contact with sea water	Structures near to or on the coast
XS2	Permanently submerged	Parts of marine structures
XS3	Tidal, splash and spray zones	Parts of marine structures

3.9- Analysis of particular prestressed structural member (Clause 5.10):

3.9.1- Generalisation to Eurocode 2:

1) The prestress considered in this Standard is that applied to the concrete by stressed tendons.

2) The effects of prestressing may be considered as an action or a resistance caused by prestrain and precurvature. The bearing capacity should be calculated accordingly.

3) In general prestress is introduced in the action combinations defined in EN 1990 as part of the loading cases and its effects should be included in the applied internal moment and axial force.

4) Following the assumptions of (3)) above, the contribution of the prestressing tendons to the resistance of the section should be limited to their additional strength beyond prestressing. This may be calculated assuming that the origin of the stress/strain relationship of the tendons is displaced by the effects of prestressing.

5) Brittle failure of the member caused by failure of prestressing tendons shall be avoided.

6) Brittle failure should be avoided by one or more of the following methods:

   Method A: Provide minimum reinforcement in accordance
   Method B: Provide pretensioned bonded tendons.
   Method C: Provide easy access to prestressed concrete members in order to check and control the condition of tendons by non-destructive methods or by monitoring.

Method D: Provide satisfactory evidence concerning the reliability of the tendons.

Method E: Ensure that if failure were to occur due to either an increase of load or a reduction of prestress under the frequent combination of actions, cracking would occur before the ultimate capacity would be exceeded, taking account of moment redistribution due to cracking effects.

Note: The selection of Methods to be used in a Country may be found in the National Annex. The UK National Annex accord any of the method A to E can be used.

4- Temperature Effect

4.1- INTRODUCTION

The temperature effect is major for prestressed concrete because the stress from thermal actions can modify the volume and/or help to create crack in the concrete. The temperature appears has different levels of the design of prestressed concrete. The temperature affects for example the creep and the shrinkage of the concrete during the early age (explain in the creep and shrinkage review) of the structures but also affect the structure has long term. The variation of temperature along a beam called gradient thermal can produce differential stress, producing internal efforts. More technically the changes in temperature can set off additional deformation and stresses and if they are significant, it can affect the ultimate and serviceability limit states of structures. This part of the project describes principles and rules provided by the Eurocodes and by pass researchers papers.

4.2- THE EUROCODES AND TEMPERATURE EFFECT

4.2.1- Scope and fields of application to the Eurocode 1: Part 1-5: General actions _ Thermal actions

This part of the Eurocode presents the principles and rules for calculating and design thermal actions on buildings, bridges and structural elements. Thermal actions on a structure or a structural element are those actions that arise from the changes of temperature fields within a specified time interval, (En 1991-1-5, §1.5.5). This part of the Eurocode gives moreover a description of the changes in the temperature of structural elements. In addition this part gives the characteristic values of thermal actions, which are applied on the design of structures exposed to daily and seasonal climatic variations. Structures with thermal actions as their main function are also treated for example chimneys, tank, etc… Furthermore the characteristic value of isotherms of national extremes of shade air temperatures is given in forms. The shade air temperature is the temperature measured by thermometers placed in a white painted louvered wooden box as a “Stevenson screen”.

The design philosophy adopted in Eurocode 1: Part 1-5 states that the temperature distribution in a cross-section leads to a deformation of the element. If the deformation is restrained, the stresses in the element may occur. Part 1-5 needs load-bearing actions on structure to make sure the thermal actions do not produce overstressing into the structural elements. And also to provide expansion joints or to take account the resulting effects in the design.

Classification and representation of thermal actions

Thermal actions are classified as variable and indirect actions. The design situation is in accordance with Eurocode 0. The characteristic values are usually 50-year return values. The representation of actions considering are daily and seasonal changes in shade air temperature, solar radiation, etc. The magnitude of the thermal effects depends of the local climatic conditions, the orientation of the structure, its overall mass etc. The temperature distribution is divided to four components, firstly a uniform temperature component, a linearly varying temperature component about the z-z axis and the y-y axis and a non-linear temperature distribution. A uniform temperature component is the temperature constant over the cross section, which rules the expansion or contraction of an element. (En 1991-1-5, §1.5.7)

Temperature changes in building

For temperature changes in buildings, Part 1-5 gives a guideline and advice but no detailed design rules. The common habits is to investigating the effects of thermal actions on building structures and cladding where the temperature in the envelope usually changes by less than 20°C during normal use. The cladding is the part of the building, which gives a weatherproof membrane, and usually cladding will only carry self-weight or wind actions, (En 1991-1-5, §1.5.6). The effects of the thermal actions are considered if there is a risk of being over the ultimate or serviceability limit states because of the thermal movement. In addition it advises that differential movement involving components formed from various materials should be taken into consideration. Allowance for differential movement between the structure and cladding and between the cladding components is needed.

Temperature changes in bridges

For temperature changes in bridges, Part 1.5 of the Eurocode 1 splits the superstructures into three different groups. Group 1 is about steel deck on steel box and truss or plate girders. Group 2 is about concrete deck on steel box and truss or plate girders and finally group 3 is about concrete slab or concrete deck on concrete beams or box girders. Part 1-5 advice to take into account the characteristic values of the vertical linear temperature component, which is given by the over a period of time, cooling and heating of the upper deck's surface of the bridge will result a maximum temperature variation in the top and bottom surface, this mean the thermal gradient. These values are established inside a normative annex for the assessment of non-linear thermal actions in bridges.

Temperature changes in industrial chimneys and pipelines

At last the Part 1.5 has a section, which gives experimental values on temperature changes in industrial chimneys and pipelines, because of the variation of shade air and solar radiation. It also needed values from the project specification on the operating process temperature. There is different thermal actions defined depending if the structure is in contact with heated gas flow or heated material (accidental temperature distribution from failures in operation or temperature distribution for normal process conditions). The characteristic values of maximum and minimum flue gas temperatures are not known inside the Eurocode 1, because the project specification must provided it for a 50-year return period. For these structures the thermal action measured are uniform temperature component of the temperature, the linearly varying temperature component and also the solar radiation producing a stepped temperature distribution round the structure's circumference.

Eurocode 2 and temperature effect

Eurocode 1, Part 1-5 is intended to be used with Eurocode 0 for the design and combinations of actions. It is also intended to be used with the other Part of the Eurocode 1.The design of the prestressed concrete is executed with the guidance of Eurocode 2. This means if there are thermal actions calculated with the help of Eurocode 1, Part 1-5, the thermal actions will be include inside the design of Eurocode 2.

The section basis of design and actions and environmental influences to the Eurocode 2, (En 1992-1-1, §2.3.1.2) display the requirements for the thermal effects. The thermal effects must be taken into account during the examination of the serviceability limit states and be considered for the ultimate limit states, the failure when they are important (e.g. fatigue condition). The Eurocode 2 necessitates accounting the thermal effects as variable action. This means a partial factor will be applied on it.

The thermal actions on the elastic deformation of the concrete consider a linear coefficient of thermal expansion equal to 10.10^{-6} $K^{-1}$for traditional concrete, (En 1992-1-1, §3.1.3 (5)) and 8.10^{-6} K^{-1}.for lightweight concrete, (En 1992-1-1, §11.3.2). For the linear elastic analysis, the thermal deformation effect is influences the ultimate limit state as a reduced stiffness corresponding to the cracked sections, neglecting tension stiffening but including the effects of creep. It also influencing the serviceability limit state as a gradual evolution of cracking, (En 1992-1-1, §5.4 (3)). For the estimation of the deflection, it is considered that if there are thermal action the flexural tensile strength of the concrete is equal to $f_{ctm,fl}$, (En 1992-1-2, §7.4.3).
In the structural analysis, it explains that beneficial effects of horizontal restraint produced by friction because of the weight of any supported element can be applied where the bearing arrangements prevent the possibility of permanent growing sliding of the elements, such as initiated by irregular behaviour under alternate actions as cyclic thermal effects on the contact edges of simply supported elements, (En 1992-1-2, §10.5.1).

The thermal effect is one of the causes for shrinkage, creep, relaxation and deformation. These effects are all causing loss of prestress. The shrinkage and creep have with the same kind of requirement as thermal effects (En 1992-1-1, §2.3.2.2) and also the deformation of concrete due to the action and environmental influences. (En 1992-1-1, §2.3.3).

4.3- WHY THE TEMPERATURE EFFECT IS SO IMPORTANT IN THE DESIGN?

Changes in temperatures may cause additional deformations and stresses and may, in some cases, significantly affect ultimate and serviceability limit states of structures. Fundamental principles and rules described in this part of the project provide basic tools for specifications of temperature changes and for the evaluation of thermal actions effects in buildings, bridges and industrial structures.

4.4- RESPONSE OF THE CONCRETE TO THE TEMPERATURE VARIATIONS

Deformation due to temperature may appear without external force. The displacement or expansion might create a crack. This research project is to find out how to prevent, design and calculate this possible deformation and annul their effects for example the cracks and failure. It is understood the loss of prestress in a prestressed concrete member is signification of failure and collapse. The deformation is possible because of the effect of the variation of temperature can create a possibility of creep of the concrete over the prestressing tendons and let the cables relax, with inevitably loss of the prestress.

4.5- RESPONSE OF THE STRUCTURAL MEMBER TO THE TEMPERATURE VARIATIONS

In a structural member the variations in temperatures distribution may be illustrated as a uniform component and a temperature or thermal gradient. Changes in average uniform temperature vary just the axial length of a members and the temperature gradient produce bending deformations. It is considered that these changes must have the capacity for deformed because the entire structure can suffer of longitudinal expansion causes lateral displacement and larger bending moment This deformation is comparable in concept to the secondary moments caused by prestressing.

Above the height of a structural member, the temperature distribution may be non-linear. Those bring on thermal stresses, which occurs during curing and in service. The relative intensity of the thermal stresses that contribute to the concrete stress depends on the structure properties and the temperature distribution. Nowadays the intensity of stress for a number of structural members can be resolved by computing for example a girder (P. J. Barr; J. F. Stanton; and M. O. Eberhard 2005)[12]

The temperature effect is often more pronounced on prestressed concrete beams as any other structures. The temperature profile through the depth of the beam (Emerson 1973[28]) can be divided into three components for the purposes of calculation. (Hambly 1991[29]) The first causes a longitudinal expansion, which is normally released by the articulation of the structure, the second causes curvature which leads to deflection in all beams and reactant moments in continuous beams, while the third causes a set of self-equilibrating set of stresses across the cross-section. The reactant moments can be calculated and allowed-for, but it is the self-equilibrating stresses that cause the main problems for prestressed concrete beams. These beams normally have high thermal mass, which means that daily temperature variations do not penetrate to the core of the structure. The result is a very non-uniform temperature distribution across the depth, which in turn leads to significant self-equilibrating stresses. If the core of the structure is warm, while the surface is cool, such as at night, then quite large tensile stresses can be developed on the top and bottom surfaces. However, they only penetrate a very short distance into the concrete and the potential crack width is very small. It can be very expensive to overcome the tensile stress by changing the section or the prestress, and they are normally taken into account by the provision of a mesh of fine bars close to the surface. A larger problem can arise if thermal stresses act as a trigger for more damaging cracking, such as the release of locked-in heat of hydration effects which can occur when a thick web is associated with thin slabs.

4.6- SOLAR RADIATION

Solar radiation is when the sun affects an open surface by encouraging rise in temperature. There are two effects due to solar radiation. Firstly, the top surface expand in comparison to the bottom moving to upwards deflection at mid span, the opposite phenomenon happen in the winter when the surface et cold. It is a small deformation and can normally be mistreated. The next effect is very important; it is when the temperature is rising from the ambient (shade) temperature inside the whole section and this augment the linear expansion. This phenomenon is especially significant is multi-storey structure such as car park or open black surface.

.

4.7- THERMALS GRADIENTS

Bridge (Le Delliou, 2004 [1]) beams are under the phenomenon of outside temperature. The top part of the beam is subject to the sun effect (solar radiation), which heats by sunrays. It results in this part being more hot as the surrounding temperature and the bottom part is colder to the surrounding temperature. Furthermore the road on the deck is usually in black asphalt and the dark body attracts the sunrays, which increases the temperature.

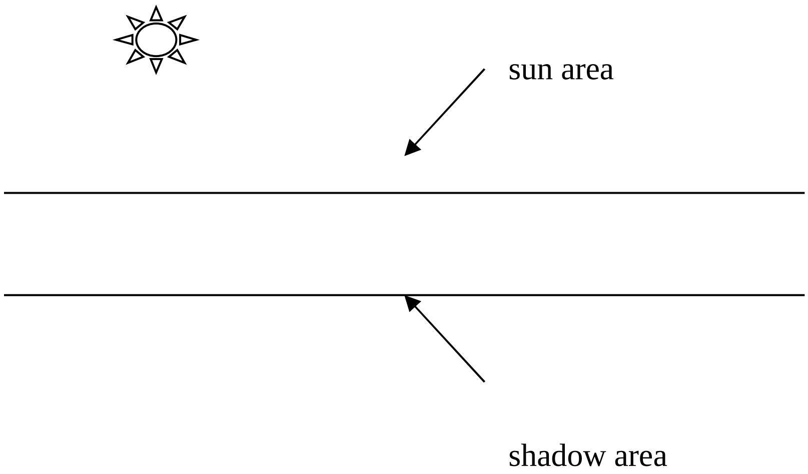


Figure 4.1: heats on the bridge deck

Considering a rectangular beam with constant height h and length l. Also taking the temperature distribution law over the height of the beam being a linear function of ordinal y. Thermal gradient is the name given to the difference between the temperature θ of the top and bottom fibre.

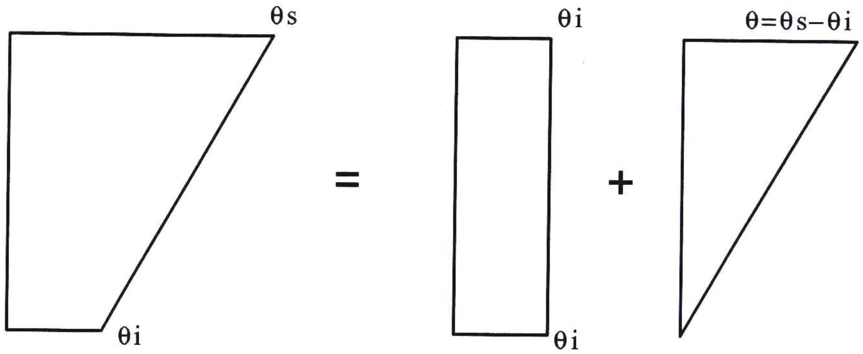


Figure 4.2: Linear variation of temperature in a beam

Under the effect of the temperature the top fibre get longer of $\delta lt = p \times l \times \theta t$ and the bottom fibre of $\delta lb = p \times l \times \theta b$, p is the coefficient of dilatation of concrete, about 10^{-5} K^{-1}.

If the distribution of the temperature is linear, a fibre located from the ordinal y from the bottom of the beam has a different strain as the bottom fibre of $\delta l\ (y) = pl\theta y/h$.

The variation of elongation being constant in the length of the beam, the beam has a deformation in arc with the deflection to the top. If the beam is isostatic, the deformation can be done and do not produce new parasite stress.

However if the beam is hyperstatic, the extra connections block the deformation and create a new distribution of the force. Example a beam over three supports. Under the thermal gradient the beam curve and rise up from the middle support.

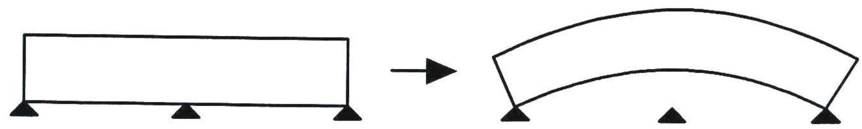


Figure 4.3: Deformation under positive effect of thermal gradient

To oblige the beam to stay in contact with the middle support, it is necessary to apply a vertical force F directed from top to bottom. This force is balanced by the two force directed from bottom to top in the exterior support. If the two segments are the same, R0 and R2 are equal to F/2.

The beam is subject to a bending moment which in the any section located in the left segment of the beam is equal to $M\ (x) = F \times x/2$.

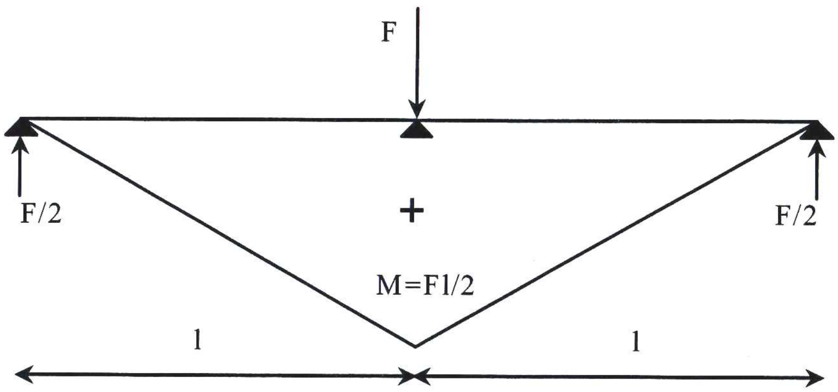


Figure 4.4: Hyperstatic moments due to thermal gradient

The design of the forces causing by the thermal gradient can be done by using the following relation:

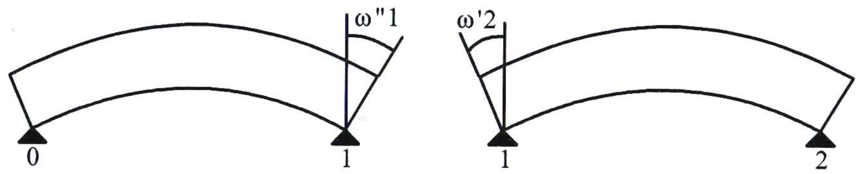


Figure 4.5: Deformation in a segment becoming isostatic

$$\omega'2 = -\,\omega''1 = -\frac{l}{2R} = -\frac{l+\delta l}{2\times(R+h)} = \frac{\rho\times l\times\theta}{2h}$$

The beam segments being with a constant inertia, we establish:

$$M1 = 3\times E\times I\times\frac{\rho\times\theta}{2h}$$

The reaction in the support due to the thermal gradient is:

$$R0 = R2 = -\frac{R1}{2} = 3EI\frac{\rho\times\theta}{2hl}$$

Comments: This phenomenon has been one of the factors (for prestressed concrete) that in the past made failed bridges. The most influence by this phenomenon is the box beam.

The Eurocodes gives two cases of thermal load corresponding to linear gradient of +15°C and –8°C. The forces are normally pondered and added to the ELS combination.

In reality, the distribution of temperature is not linear but it looks like the figure 4.6: for box girder.

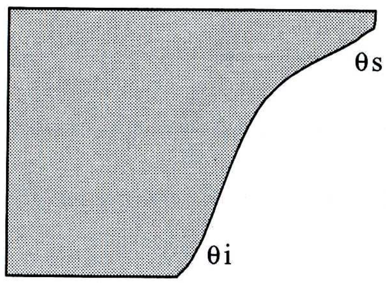


Figure 4.6: True temperature distribution

Studies show that the difference between the linear distribution law and the true distribution produce stress irrelevant for the concrete and can be ignore.

4.8- TEMPERATURE RANGE

To design the temperature effect, engineer needs daily, seasonal temperature and also the thermal expansion coefficient of the concrete and to understand the solar radiation effect leading to linear expansion and finally the temperature range. It is casting temperature articulated as a 2 hours indicate ambient temperature, as the rise over ambient and drop back to it have been taken in consideration for early thermal contraction. Casting happen at any time and any temperature over 5°C.

4.9- CONCLUSION

The combination of the movement discussed above happens at any time and cause the linear expansion or contraction. Initially, contraction in early days will lead but creep of concrete relatively relieves it. In the first summer of the structure cast in winter, the temperature is rising and expansion happens. It is often inconsequential because the deformation will create only very small movement. But when the seasons changes and especially for structure cast in summer, drying shrinkage will occur and temperature will drop and the global effect will be contraction. This is significant because tension is bringing on and cracking may take place.

There are different ways to study the effect of temperature. The structural member can be in an extreme temperature, for example very cold in the ice or very hot during a fire. The structural member can be under a variation of temperature short or long term and finally the structural member can inside is thickness be under variation of temperature. Example a chimney for a steel company where outside the temperature can be low and inside the chimney the temperature can go to 300°c. The temperature effects over concrete are omitted in the global analysis inside the Eurocode 2, which have a preference to using other material techniques for this. The temperature effects over prestressing steel relaxation uses a temperature of 20°c after 1000 hours of tensioning.

The temperature has a major effect on concrete structure and specially prestressed concrete. For the reason that are any deformation in prestressed concrete member has for result to producing other deformations effect such as shrinkage and creep of concrete but also crack and failure of structural member. The calculation of the thermal gradient for the temperature distribution in prestressed concrete member is significant for any structures but mainly for heavy thermal mass for example bridges decks, which is subject easily to deformation such as deflections and also stresses when the movement is restrained. Finally I advise the engineer during design to check the thermal gradient distribution.

5- CREEP AND SHRINKAGE EFFECT

5.1- INTRODUCTION

Creep and shrinkage effects are related and they have been studied very often together. In this part it explains the phenomenon of creep and shrinkage through the knowledge understood during the last 30 years from the International Symposium on Fundamental Research on Creep and Shrinkage of Concrete in Lauzanne (September 1980) to the newest studies. It has been considered since long time but the comprehension phenomenon and the consequence on concrete structure are relatively recent. It is with the ability of powerful computers that researchers and engineers handle more easily the mechanism by creation of mathematical model and using finite element. This part gives the conclusions of the more important research done by the best specialist of these both effects. It is still not used in current practice because the international codes and recommendations are relying on simplified material laws.

5.2- CREEP AND SHRINKAGE MECHANISMS

Wittmans[44] classified several shrinkage mechanisms. These mechanisms have been subdivided into three distinct groups. The first group is the capillary shrinkage. The water between two particles is under capillary pressure because the particles are attracted to each other, this happens when the concrete is in a fresh state and the surface is drying or when the drying proceeds suddenly and water is trapped in the concrete, between the particle so the concrete will shrink few hours later. The second group is the chemical shrinkage. This means the volume is changed because of chemical reactions. One of these chemical reaction is the hydration of cement, because the cement react with water the volume of water add to the volume of cement is more important the volume cement/water. Hydration shrinkage happens usually two hours after the blend. The problem with water is after this time known as capillary shrinkage. Other chemical reactions can happen as thermal shrinkage when the rate of hydration slows down the temperature decrease in the concrete, which may cause serious cracking. Also the crystallization shrinkage when the hydration is rapid the water can be trapped and when the solid skeleton of the concrete is built up further crystal growth is delayed and thus creates an internal pressure. The author gives an explanation for the carbonisation shrinkage. He explains the phenomenon, as during hydration of cement, there is a reaction with the CO2 from the surrounding air and that under normal climatic condition this happens in the surface of the concrete. When the water is liberated and evaporates this creates a decrease of volume. The last group is the drying shrinkage known as wetting swelling and can be defined as changes of volume when the moisture contents changes.

Wittmans[44] classified two creep mechanisms. The first mechanism is the short-time creep, which may be described by a redistribution of water in the microstructure because of a stress such as capillarity pressure in the structure. The second mechanism is the long-time creep, which may be described by the displacement of gel particle when the porous system adsorbed the capillary condensed water.
Studies show that the Creep and shrinkage deformation is shown schematically as a function of time. When creep and shrinkage appears simultaneously, the deformation is higher than the sum of creep and shrinkage when they are measured separately but the interrelation between the both are not well know and have to be more discuss. Also a number of apparent mechanisms are always involved and they modify the behaviour.

5.3- GENERAL ASPECT OF THE CREEP AND SHRINKAGE IN CONCRETE STRUCTURE

5.3.1- The microstructure of hardened of cement paste

Traditionally the structure of the hardened Portland paste is detailed as an intimate but heterogeneous mixture that has various elements with generally varying properties and characteristics. The cement components are described and treated separately. Diamond[43] illustrates three basic phases of microstructure. The atomic level, which is involved crystal and molecular structure; the particle level, which is mainly involved morphological the aspects of crystal and particles; and the micro morphological level, which is explains how all components suit together. Nevertheless, in view of creep and shrinkage phenomena Young [39] describes another useful approach to take into account microstructure in terms of a unique component. These micro structural components are difficult to characterise but that make the understanding of creep and shrinkage phenomenon better.

5.3.2- Used of mechanics of concrete systems and current approaches to assessing material behaviour to models creep and shrinkage.

One way to comprehend the creep and the shrinkage is to understand the behaviour of the concrete systems. Dougill[40] has examined the different procedures from a structural analysis point of view to describe the behaviour. For the study a conventional stress and strain and material properties are used in the analysis connected to the significant variables such as time and temperature. The procedures to describe the concrete behaviour are models, which are developed a range of phenomena and circumstances to a single material. There are two mains models, which are the non-linear behaviour, including breakdown, failure, degradation or the linear models including elasticity, creep, shrinkage and thermal effect.

For this project I overview the linear model called self-consistent models. This model relies on assumption concerning the average stress or strain. It is believed the stress average and properties are similar in a single particle as a composite material. These procedures allow to obtain expression for the moduli and to study elasticity. Dougill[40] used the self-consistent approach to determine the thermal expansion, the shrinkage of concrete and the creep. This approach has given the opportunity to fit physical processes into a simple structural context and to extent the models to a range of new phenomena. It is also demonstrating that models can be linked together. This means that the effect on one factor affect the others factors and by consequences the models get complex. The models must be flexible to let different modes of behaviour as shrinkage or creep to be included.

5.3.3- Time-dependence:

For his research on the probabilistic approach to deformation of concrete, Cinlar[41] gives an idea of the role of the water and the temperature on the rate of creep. Indeed his aim was to develop a model of time dependant of deformation of concrete set up on microstructure. As concrete blend aggregate and sand embedded in a matrix of hardened cement paste. This matrix is very porous and has for major component the cement gel. The pores are two types. The Macrospores, largest pores with round shape and contain capillary water. They are connected by Micropores, thinnest pores with laminar and tubular shapes and contain absorbed layers of water molecules and solid particles.

5.3.4- Migration function

If a load is affected to concrete, the majority of compression in the laminar micropores is transfer by solid particles. For one position of micropore, the result of high enough transverse pressure is to press on some of the bonded particles over their activation energy barriers. They are producing them to travel to position of lower stress. The transverse pressure can be reduced and this can stop the migrations at that position for a moment. At the same time, there is a loss of mass and thickness in the adsorbed layers, and the loss of thickness contributes something to the total deformation. If the migration is going through the position, these mechanisms are inverted. The term ‘migrations’ can mean two connected events, which effects on deformation have, opposites. Firstly, the bonds division and the de-bonded particles movement. Secondly, the new bonds of particles formation, which restrain additional movements and slow down local deformations.

5.3.5- Water function

The migrations need the presence of water molecules, without them there is almost no creep. The rate of creep is higher when the number of particles migrating is higher which is influence and bigger when the water content is higher. If there is a variation in the water content (in any direction), a lot of water molecules move to the micropores, and they are causing to the solid particles larger mobility and greater creep rate. Although that there is no load, the phenomenon will create some creep.

5.3.6- Temperature function

Usually if there are elevated levels of energy it is because of high temperature. This affects the particles and facilitates to jump over their activation energy barriers, water solid particles get more mobility when the temperature is higher. Consequently the effect is greater migrations and greater creep rate at the macroscopic level but this causing faster aging which one after another decrease the rate of creep.

5.4- CONCRETE DRYING AT DIFFERENT HUMIDITY AND TEMPERATURES:

Research on the estimation of drying of concrete at different relative humidity and temperatures of ambient air with create special discussion about fundamental features of drying and shrinkage. The author Pihlajavaara[42] explains that the drying porous solids are a complicated physicochemical hygrothermal phenomenon and present fundamental features of drying and shrinkage of concrete. In the history of concrete the first phase is the diminution of moisture content and that gives a relevant increase strength and creep (drying creep) and it is the major cause of shrinkage (drying shrinkage). The authors said these changes are almost irreversible, which means the concrete is a history-dependent material. In the microstructure component a slow uptake of CO2 from ambient air are making variation and carbonisation shrinkage in the surface layer. It is also means the drying and shrinkage is interrelated factors for the deformation of concrete. This study shows that the concrete carbonated and the concrete non-carbonated do not have similar final shrinkage. The carbonation gives an extra shrinkage called carbonation shrinkage, which adds to the drying shrinkage. It has been estimated the carbonisation is limited when the conditions are very dry or very wet. The temperature is also an important factor here. The study demonstrates from an experimental result that the effect of the water/cement ratio on shrinkage is related. The shrinkage increases with decreasing W/C and the shrinkage increases with increasing W/C. On this study we can say that there are definitely an interrelated connection between factors like shrinkage, temperature, drying and carbonisation and water/cement ratio.

5.5- RESULTS OF THE MEASUREMENT AND MODELLING OF CREEP AND SHRINKAGE EFFECTS

Many different factors are needed and not only the applied load to understand the time- dependent deformation of concrete. The factors are usually the strength, the maturity of the concrete, the moisture and temperature histories experienced by the concrete. It is also the materials properties of the concrete, aggregate properties, the size of the concrete structural member. Actually the experimental techniques that are utilized studies of the shrinkage and creep of concrete must be selected as much for the way the environmental parameters may be controlled and changed as for the methods of load control and strain measurement. The measurement of creep and shrinkage can be done on hardened cement paste because it is the constituent, which contributes the most to the time-dependent dimensional changes. But also the measurement can be on concrete in laboratory or on site to understand the actual behaviour of concrete in a structure. From my point of view this research has to be taken with precaution because the unknown factors are high and a basic understanding of the causes of creep is necessary for used wisely the concrete on any circumstances.

5.6- PHYSICAL ORIGINS OF SHRINKAGE

Researcher classified three physical origins of shrinkage. The mechanical actions due external forces and displacement as boundary condition like settlement. The thermal actions which are come from in one hand the natural origin (climate) or industrial origin (thermal treatment to accelerate concrete hardening) and in other hand the heat produced in the concrete by hydration of the cement. Finally the Hygral actions causing by the properties of the concrete. The concrete being a porous material, capillary pressure compressed the solid skeleton.

5.7- CREEP AND SHRINKAGE INTERRELATION

Some literature evaluates creep and shrinkage together. In their research for the evaluation of a basic creep model with respect to autogenous shrinkage, *Yun Lee, Seong-Tae Yi, Min-Su Kim ,and Jin-Keun Kim*[32] *found that the following conclusion.*

- *Autogenous shrinkage is absolutely present and not insignificant for low W/C concrete and at early age of normal strength concrete (NSC).*

- *Clear differences happen in the apparent and real creep conform functions at early stages of loading. The meaning of these differences increases as the w/c of the concrete decrease.*

- *Autogenous shrinkage and basic creep deformation test must be separated for analysis the numerical stress or the thermal stress analysis and crack control problems.*

Furthermore, the latest research on the interaction between early-age deformation factor as drying, shrinkage, creep and cracking phenomena in concrete and base on Bazan [45] research tried to improve the accuracy of the result by understanding the effect of the factors like the role of the water to the creep. Benboudjema, Meftah and

Torrenti[33] highlight the fact that the water is influencing every single factors of deformation and also interrelated together the factors in their basic behaviour.

5.8- PREDICTION OF CRACKING AT EARLY-AGE CONCRETE DUE THERMAL, DRYING AND CREEP BEHAVIOUR

Research has been done on the prediction of cracking within early age due thermal drying and creep behaviour. To predict the early age cracking, numerical simulation based on micro mechanical model have been realised to understand the property development of young concrete. In this research, factors as environment influences, creep, hydration and moisture have been take into account. The authors takes for is approach microscopic stress as the water vapour, capillaries and also heat and moisture diffusion. The authors (Y.Yuan; Z.L.Wan 2002 [34]) describes that the temperature and the humidity distribution can be translated and changed as extra loads to implant to the structure and that will affect the creep effect and the elastic analysis of the concrete behaviour such as stress and strain. For the calculi of the creep the additional loads gives an independent deformation response to the past load response. This mean, there are a superposition of principal and the traditional interpretation of the concrete must been subdivided from time interval to time multisteps.

Research on the investigation on key properties controlling early age stress development (factors controlling early-age cracking) of blended cement concrete show that the temperature effect is an important repercussion factor on creep. Pane and Hansen [35] during the investigation describe the age dependent behaviour of creep and shrinkage as functions of heat of hydration, which create stress in early-age. And the heat of hydration is depending from the temperature variation inside concrete and the ambient temperature variation.

5.9- RECAPITULATIVE OF SHRINKAGE

Definition :

Depending the mechanism of concrete shrinkage, which can be a plastic shrinkage, an autogenous shrinkage (process known as self desiccation), drying of shrinkage and carbonation of shrinkage.

- Autogenous shrinkage is the result of extraction of the water from the capillary pores by the anhydrous cement particles. The autogenous shrinkage happens usually during the early stage of hydration of cement, otherwise the phenomenon happens when the concrete mixture has a very low water/cement ratio and if the moisture is available.

- Plastic and drying shrinkage are due to the removal of the water from concrete under the condition of humidity gradient between the interior of concrete and the air. This two phenomenon are factors for the cracking of concrete in early age.

- Carbonation shrinkage is due by the carbonation of calcium hydroxide in the concrete. This phenomenon appears generally of the surface of concrete members.

Significance of studying shrinkage of concrete

Shrinkage of concrete is a major behaviour for the property of concrete. The phenomenon influences the cracking even the failure. At the early age the poor strength in the concrete may not contain the stresses from drying shrinkage, which causes cracking in the concrete. And the shrinkage cracking in the concrete can afterwards run premature failure in the structure and corrosion of the steel in structural members. For the prestressed concrete members not only the shrinkage inducing cracking and then corrosion the steel can be a problem. Also the shrinkage deformation that directly influence the prestress loss.

5.10- RECAPITULATIVE OF CREEP IN CONCRETE

Definition of creep:

If you test concrete under loads for a strain test, and after you de-loaded the concrete the strain remaining is the creep strain. The creep strain can be divided into two parts, basic or drying creep.

- True or basic creep happens when the concrete is under the condition of no moisture movement to or from the ambient medium.

- Drying creep is due to drying of the concrete.

Significance of studying creep of concrete

Creep is a major phenomenon for the concrete behaviour. It can be a positive and a negative property. The main positive action from the creep is that it can relieve stress concentration due to shrinkage, temperature changes and the movement of support. If a beam is fixed at both ends, creep deformation will influence the reduction of tensile stress caused by the shrinkage and the temperatures changes.

The negative action from creep is to damage the safety of a structure by running the structural members to excessive deflection, buckling and other serviceability problems. When a cycle of temperatures changes happen in the mass concrete, because of the heat hydration development. This phenomenon results as creep cracking. For prestressed concrete structures the creep of concrete must be the lower possible. The members who are heavy and long are especially susceptible to large volumes changes. It is important to allow flexibility to the connection for a prestressed concrete member. If not, the volume changes will create stress on its connection and support, which can leads to structural failure.

5.11- REDISTRIBUTION OF THE CREEP FORCES

The concrete is not an elastic material. Under the stress σc the concrete undergoes an instantly deformation εi and with time a postpone deformation εv which is usually about twice εi. This propriety is important consequences for the geometry of any structural members. Further this propriety changes the forces in the structures and can be a problem for by example the structures construct by phases.(Le Delliou 2004 [1])

Effect of the creep on the geometry

Considering a beam on two supports. The beam is prestressed by a tendon thus looks like the figure 5.1.

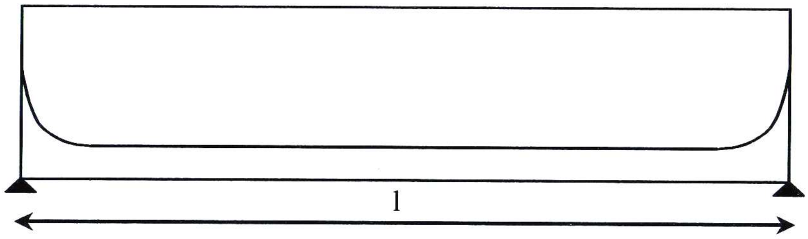


Figure 5.1: Prestressed beam by an eccentric cable

In most of the beam the tendon is close to the bottom fibre. During no loadings, in quasi-permanent time, the compression stresses of the concrete are small in the top fibre and strong in the bottom fibre. With time the bottom fibres cut down a lot more than the top fibres and the beam has a bending deformation in height direction.

If for example we study a beam of 40-meter length and 2.20m high. The beam is subject to the creep stress only, on the top fibre $\sigma ctop = 0$ and on the bottom fibre $\sigma cbot=14$ MPa.

The concrete moduli is $Ec = 40000$Mpa.

Under the creep effect, the bottom fibre get shorter by $\delta l = 2\sigma cbot \times l / Ec$ however the top fibre keep the same length.

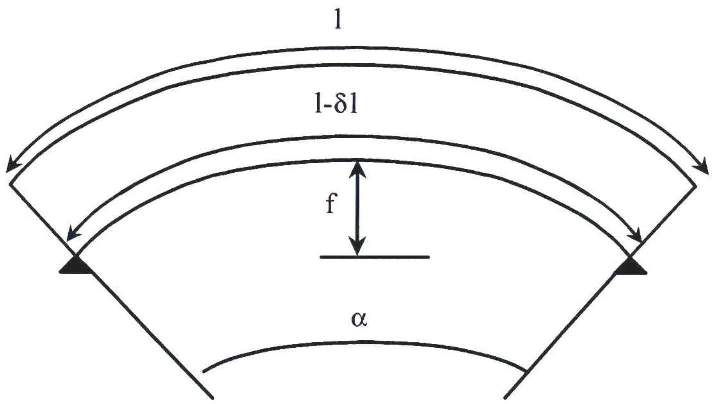


Figure 5.2: Beam deformation under the creep effect

In this case $\delta l = 2 \times 14 \times 40 / 40000 = 0.028$ m.

The flexural deflection f is equal to:

$l = \alpha R$ $\qquad$ $l - \delta l = \alpha(R-h)$

$R = l \times h / \delta l$ $\qquad$ $\alpha = \delta l / l$

In this case $R = 40 \times 2.20 / 0.028 = 3142\ 85$ m and $\alpha = 0.028 / 40 = 0.7°$

$F = (R - h) - (R - h) \times \cos(\alpha/2) = (3142.85 - 2.20) - (3142.85 - 2.20) \times \cos(0.7/2) = 5.9$ cm

This is an important deformation, which happens, in some independent prestressed beam.

Beam built for its original purpose

In the case of a beam directly construct to run on its definitive state. The beam is on several support, poured on site, it demonstrates with time an increasing number of deformations from permanent loads. It is like an isostatic beam but there is no redistribution of the forces, the reaction on the support still almost constant. Indeed, the stress distribution law in the length of the beam take into account every connection applies on the structure. The creep deformation of the beam is homothetique of his deformation elastic. Considering the creep as a linear phenomenon and if we ignore the effect of the prestressed differ force. The mistake is ignorable in most of the cases. The redistribution of forces is definitely lower as the thermal gradient.

In the case of a beam not constructed to run in its definitive state. The beam is in the end of its construction, the concrete store under the effect of the self weight and the prestress, a system of stresses very different (in value and in sign) as the beam would be working in its definitive task. Under the creep effect, the beam has a deformation according its stress repartition memory. The deformation that will occur will be disturbed by the connection from the definitive state, and this will produce new forces redistribution.

This may be illustrated by an example from simple hypotheses. Concrete stress-strain law is assimilating to a linear law with a concrete modulus for a loading of long time equal to the third of instantaneous modulus. See figure 5.3.

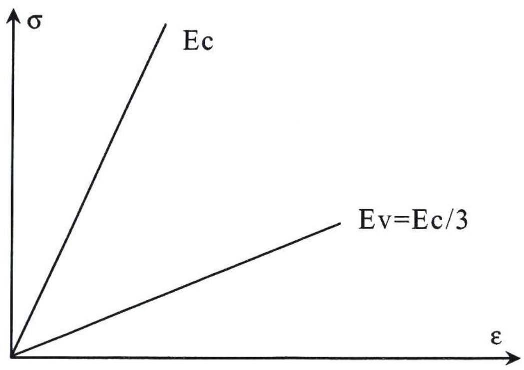


Figure 5.3: Stress-strain law instantaneous and differed

The structure is formed of beam of constant section with segment and fixed in the central column. The beam is prestressed by a central tendon which do not change the slope of the stress diagrams and by consequence, do not produce a vertical deformation of the beam during that the prestress is taking place and during time. The prestress changes are neglected and there is not creep during construction.

At the end of the construction, the beam looks like two consoles in symmetry from the column.

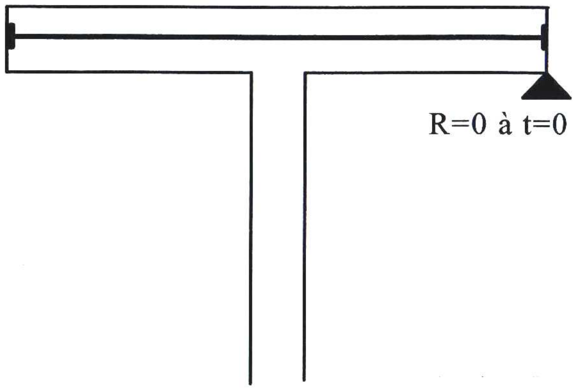


Figure 5.4: beam fixed in one support

If on the extremity of the beam is disposed of a support, which initially do not apply any forces on the beam. If p is the linear density of the self-weigh, the diagrams of the moment will be:

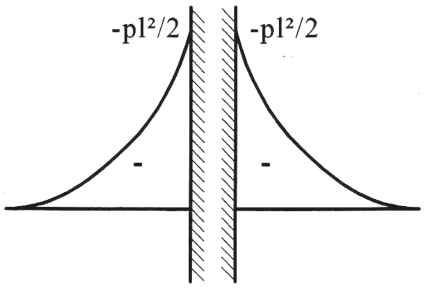


Figure 5.5: Bending moment at the end of the construction

With time the concrete creep and the beam deform. The deformation causing by the creep in the left segment of the beam is equal to the double of the elastic deformation of a console subject to a uniformly distribute load. The deflection at the extremity in the infinite time has for value:

$$f = \frac{2 \times p \times l^4}{8 \times E_c \times I}$$

Under the creep effect, the right segment of the beam would take the same deformation as the left segment, but the support prevents the deformation. The support reaction which was inexistent at the end of the construction are growing and its value in infinite time is the force R which must be apply to the extremity of the console to cancel the flexural deflection calculated before.

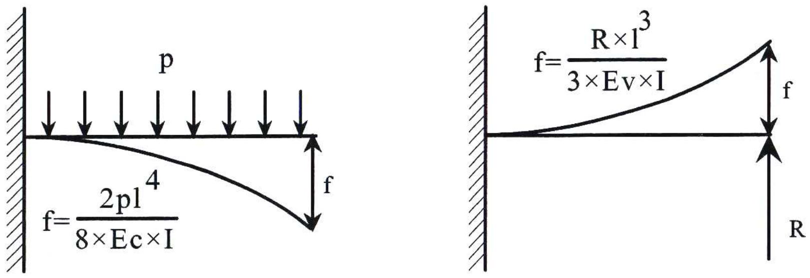


Figure 5.6: Flexural deformation under creep and support reaction

By equalling the two values of deflection, it is possible to obtain the support reaction at infinite time.

$$\frac{R\times l^3}{3\times E_v\times I}=\frac{2\times p\times l^4}{8\times E_c\times I}$$

$$R=\frac{3}{4}\times p\times l\times\frac{E_v}{E_c}$$

With the usual value $E_v/E_c=1/3$

$$R=\frac{p\times l}{4}$$

In the left segment, the deformation due to the creep did not change the forces and the moment diagrams. In the right segment of the beam, the creep increases the value of the extremity support reaction that was inexistent at the end of the construction. This is traducing by the important modification of the bending moment and shear force.

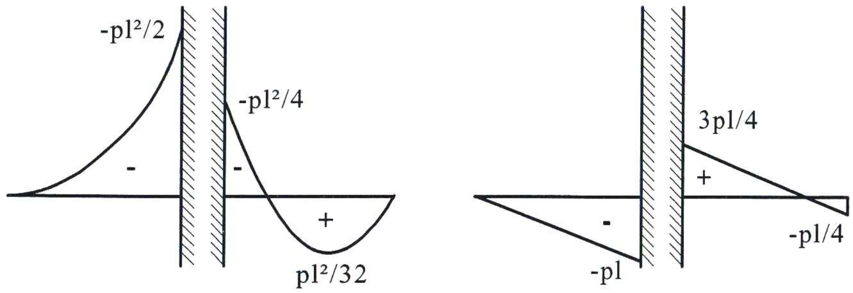


Figure 5.7: Bending moment and shear force after creep

The figures illustrate the shear and bending diagrams for the right segment in three cases:

- Beam at the end of construction
- Beam after creep at infinite time
- Beam poor on hanger in is definitive state

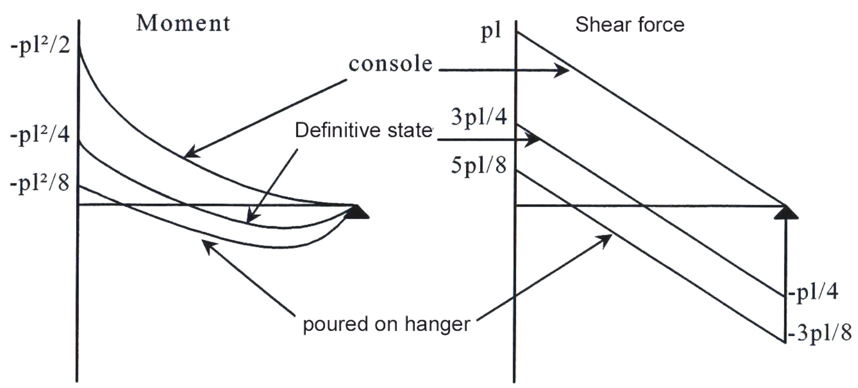


Figure 5.8: Diagrams according to three hypotheses of construction

The creep curve line is located between the two others. In any section, the bending is equal to:

$$M(t = \infty) = Mconsole + \frac{2}{3}(Mhanger - Mconsole)$$

In this example, the prestress does not influence because the prestress create only the central normal force. In the case the tendon present an eccentricity, it must be taken into account in the isostatic deformation. The hyperstatique bending of prestress happens in the calculation of the hyperstatic structure pour over hanger link.

Comments:
This example is very simplified and has a value of qualitative because:

- the concrete is more complex that the hypotheses (Concrete is not linear and not reversible) take into account and the concrete depend to lots of parameters, particularly the concrete strength (depending of the age) when the loadings;
- the prestress is not constant in time, the prestress decrease because of the effect of creep, relaxation and shrinkage;
- a part of the creep occurs before the systems are hyperstatic.

It is impossible to calculate manually, hoping to give a close approximation.

The creep being unknown and under estimate have been one of the principal causes of failures in brides construct by phases. It is usually better to consider the worst conditions:

- Take into consideration the complexity of the concrete for the theory calculation
- Take into account the half of the difference between the forces in the construction without creep and if the structure was poor on hanger link.
- Residual stress in concrete of 1 Mpa and in passive longitudinal steel of 1.5 MPa

5.12- Effects of autogenous shrinkage on post-tensioning slab

There are two major behaviours that happen to any slab with an axial compression

- Firstly, shortening caused by an axial compression happens in the slab additionally to whichever shrinkage of temperature effects. Pouring strips are used to reduce this effect because it allows short time movement but with time creep and shrinkage augment as the surrounding humidity is reducing.
- Second behaviour it is the compression in concrete. So crack are more unlikely to happens directly from either restraint forces but the axial stiffness will be higher. Consequently any forces apply to a restrain structure should be more higher.

But this effects can be positive to prevent or reduce cracking in the slab and also improve the visual point of view or the waterproofing of the structure.

5.13- MODERN TECHNIQUES TO IMPROVE PRESTRESSED CONCRETE MEMBER

Studies on the long-term behaviour of prestressed concrete members present that the effect of creep and shrinkage of concrete and also relaxation of prestressing tendons produce slow changes in the stresses in concrete and prestressing tendons. The long-term prestress loss is the main problem to the design of prestress concrete members. In the literature it has been show that using prestressing (fiber reinforced polymer) FRP tendons has a relevant influence than that when using prestressing steel. It is due to the lower elastic moduli of FRP tendons and as for consequence in the long-term changes to decrease the concrete stresses and deflection. The FRP tendons have been used for few bridges already. The advantages of FRP tendons to conventional steel tendons are for example the non-corrosive and non-conductive properties, lightweights and high tensile strength.

5.14- CONCLUSION

The interrelationship between the phenomenons is important and definitely existent. It is illustrated above that also water and temperature affects the time dependence effects in a deeper amount that the researcher believed. In nowadays every mathematical model and studies take in their analysis more and more factors to improve the design and understanding of the creep and shrinkage effects. The temperature is a special factor because it occurs at different times and for different reactions and different temperatures. During the drying, for the heat of hydration and the migration etc…, which has been more explained in the previous part of the dissertation

Research to found using stronger concrete can improved its reaction to the creep and the shrinkage. In their research for the study on creep and drying shrinkage of high performance concrete, Li Jianyong and Yao Yan[36] shows that ultrafine reduced the effect of drying shrinkage and creep. The mechanism would be that ultrafine fill the pores which means the water is blended with the ultrafine and do not create additional intern pressure in the structure.

The research on the creep and shrinkage implicate different problem and discipline and moreover for their interrelationship. The compilation of data and information from different directions motivate and contribute to a greater understanding of this both subject. The analysing and the calculation of creep and shrinkage and other effects such as steel relaxation or cable/ducts friction to Eurocode 2 for prestressed concrete design are considered in the next part of the dissertation.

Part 6- Prestress loss to Eurocode 2

6.1- DEGREES OF PRESTRESSING

The first idea of prestressing concrete was that all cracking should be avoided under service load then the section should be entirely in a permanent state of compression. It is called the "the full prestressing". Nevertheless the experimentations were still to control cracking in mix the tensioned steel with untensioned reinforcement steel in the section. It is called "the partial prestressing".

The EC2 makes the difference between uncracked (see Figure 7.12) and cracked (see Figure 7.12) concrete member in tension. Previously some tension may be authorized if the quantity is kept below the tensile strength of the concrete. Subsequently the cracks are authorised but must be limited in the width so it does not have an effect on the durability of the member. The EC2 have many clauses for the behaviour and the analysis of both types of members, which are similar.

6.2- THE PRESTRESS FORCE (CLAUSE 5.10.2):

1) P at a given time t and distance x (or arc length) from the active end of the tendon the mean prestress force $P_{m,t}(x)$ is equal to the maximum force P_{max} imposed at the active end, minus the immediate losses and the time dependent losses. Absolute values are considered for all the losses.

2) The value of the initial prestress force $P_{m0}(x)$ (at time $t = t0$) applied to the concrete immediately after tensioning and anchoring (post-tensioning) or after transfer of prestressing (pre-tensioning) is obtained by subtracting from the force at tensioning P_{max} the immediate losses $\Delta P_i(x)$ and should not exceed the following value:

$$P_{m0}(x) = A_p \,.\, \sigma_{pm0}(x) \qquad (1)$$

where: $\sigma_{pm0}(x)$ is the stress in the tendon immediately after tensioning or transfer = min $\{k_7 \cdot f_{pk}\,;\, k_8 f_{p0,1k}\}$.

note: The values of k_7 and k_8 for use in a Country may be found in its National Annex. The recommended value for k_7 is 0,75 and for k_8 is 0,85.

3) When determining the immediate losses $\Delta P_i(x)$ the following immediate influences should be considered for pre-tensioning and post-tensioning where relevant :

- losses due to elastic deformation of concrete ΔP_{el}
- losses due to short term relaxation ΔP_r
- losses due to friction $\Delta P_\mu(x)$
- losses due to anchorage slip ΔP_{sl}

4) The mean value of the prestress force $P_{m,t}(x)$ at the time $t > t0$ should be determined with respect to the prestressing method. In addition to the immediate losses given in « 3 » the time-dependent losses of prestress $\Delta P_{c+s+r}(x)$ as a result of creep and shrinkage of the concrete and the long term relaxation of the prestressing steel should be considered and $P_{m,t}(x) = P_{m0}(x) - \Delta P_{c+s+r}(x)$.

6.3- LOSS OF PRESTRESS

The tension in the prestressing steel at one point and at one instant given is different of the stress apply on the tendon by the jack from when it has taking place. Indeed, several phenomenon come adds it. There are two types of phenomenon which enter as part of the loss of prestress. The first is the immediate losses (Short-term losses, see table 6.1), this happens directly after the transfer of the prestress force to the concrete member. The second is the losses that happen gradually with time (Long-term losses).

Table 6.1: Prestress losses

	Long-term
Elastic shortening	Concrete shrinkage
Friction	Concrete creep
Anchorage draw-in	Steel relaxation

The losses of prestress are calculated with a lot of approximations. It is then not appropriate to design with great precision and with a lots of significant figures.

6.3.1- Short term losses:

In the tendon, the tension force is maximum when the tension in the jack occurs. The tension in the jack is limited in the Eurocode to minimum 0.8fpk and 0.9 fp0.1k. The problems of cable strength under regular loads occur normally at this moment. It is because of that, there is no verification of the cable strength. The tension force in the cables after anchorage is in addition, limited to the smaller of this two values, 0.75 fpk or 0.85 fpk0.1k.

The Eurocode 2 makes the difference between the pre-and post-tensioned immediate losses. The immediate losses of prestress for pre-tensioning are from clause 5.10.4 to Eurocode 2.

Elastic shortening :

The prestress force Po transferred to a structural member (i.e beam) for a pretensioned member. At the level of the prestressing tendons, the strain in the concrete must equal the change in the strain of the steel.

In the case where the tendons are closely concentred in the tensile zone the elastic shortening can be given with sufficient accuracy by taking the stress in the concrete at the level of the tendons σ_{cg} as the stress in the concrete at the level of the centroid of the tendons. In the case where the tendons are widely distributed throughout the section, then the above estimation is no longer valid, the influences of the tendons or groups of tendons ought to be determined individually and then superimposed to give the total effective prestress force.

Just after transfer of the change in strain in the tendons in the post-tensioned member, this steel strain may be assumed to be equal to the strain in the concrete at the same level, even if the ducts have not been grouted and there is no bond between the steel and concrete. Po is assumed to be constant along the member in spite of reality the force at the transfer is not constant owing to friction.

The elastic shortening is given as following:

At mid span:

$$f_{cg} = \frac{f_{po}}{m + \dfrac{A_c}{A_p(1+e^2/r^2)}} - \frac{M_0 e}{I_c} \qquad (2)$$

where:

- f_{po} is P_o/ A_p ;
- *m* is the modular ratio E_s/E_{cm} (where E_{cm} is based on the long-term value of concrete strength);
- r is $(I_c/A_c)^{0.5}$;
- *e* is the eccentricity of the tendons.

$$\boxed{\Delta_{po} = m \times f_{cg\ average} \times A_p} \tag{3}$$

<u>Friction:</u>

The post-tensioned members are the only concerned by the friction losses. The friction is situated between the prestressing tendons and the inside of the ducts during tensioning. The type of tendons and duct former modify the magnitude of the friction.

Two basic mechanisms producing the friction are:

- The curvature of the tendons to achieve a desired profile,
- Deviation between the centrelines of the tendons and the ducts

Curvature friction:
During tensioning, the tendons is tackle against the duct, in the curve part of the path, the tendon temp to put itself in straight line.

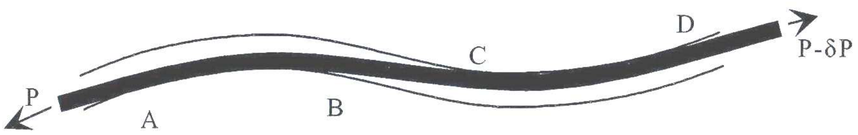


Figure 6.1: Tendon and duct relative position

There is a displacement of the tendon under the jack action, it produce a friction between the tendon and the duct, which reduce the tension in the tendon from the anchorage. A part of the force P applied by the jack is balance by the friction between A and B and between C and D (see figure 6.1) and is not pass on the extremity of the tendon.

Consider a curving fraction with a radius r and an angle at the centre $d\alpha$ (see figure 6.2). Be μ the tendon friction coefficient on the duct.

P is the tension in the tendon at the extremity of the curve. By projection, on two perpendicular direction, the total of forces on the tendon, demonstrates that the tendon applies on the duct a radial stress $p = P/r$ by length unity and an elementary friction force by length unity $t = \mu p = \mu \times P/r$.

At the extremity of the curving fraction, the duct friction on the tendon will balance a force dP such as $dP = -\mu P \times d\alpha$.

The solution for this differential equation is:

$$P(\alpha) = Po \times e^{-\mu\alpha}$$

Where Po represents the initial tension of the curve trace and $P(\alpha)$ the tension at any point located in the curve at the angular deviation α from the origin. The angle α is point given in the tendon, and it is the total angular deviation of the tendon tangent. α is independent from the exact curve between the origin and a given point, if there is absence of intermediary inflexion point. The calculation of α does not require a circular curve. In practise, if the trace is defined by an analytic function, the calculation of α will be done from the derivation of the tendon curve trace equation.

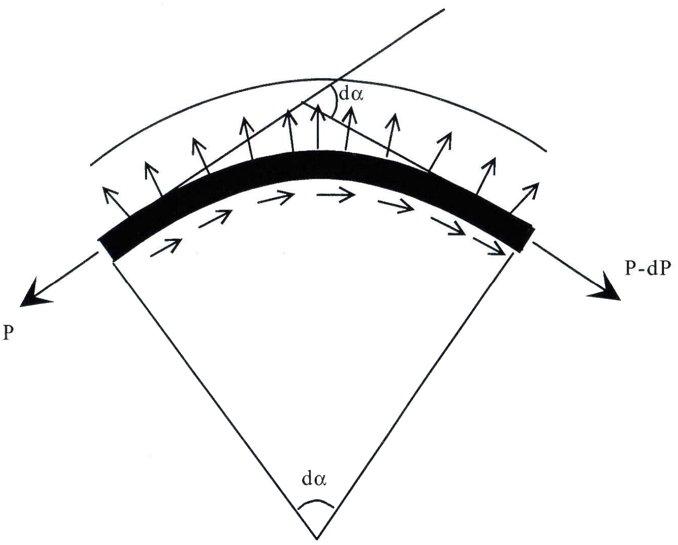


Figure 6.2: Curve reaction and friction

If the tendon present several curve, α represent the total of arithmetic deviation.

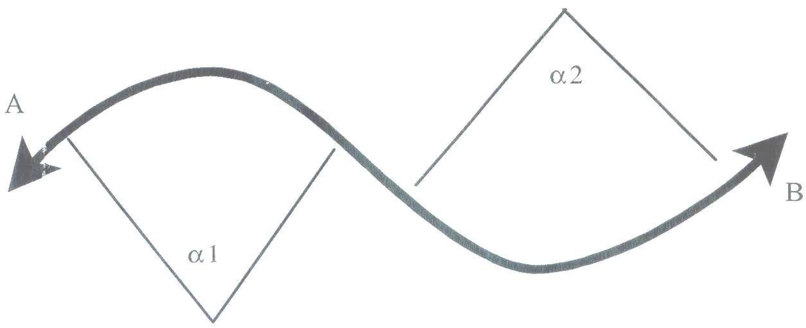


Figure 6.3: Successive deviations

The force in B is written:

$$P_B = P_A \times e^{-\mu(\alpha 1 + \alpha 2)}$$

In the case where the tendon has successive curve in both direction, α represent the total of arithmetic deviation in the both direction between the origin of the tendon and the point where the calculation of the tension. If the tendon bends, α is calculated equal to the sum of αx and αy, this method over estimates the true deviation but also major the loss.

Parasites deviations

In the area where the duct is straight, there is in theory no contact between the cable and the duct so there is no friction. In practise, a duct is not rigorously straight. It can present some imperfection more or less important according to the execution quality. The trace is close to figure 6.4.

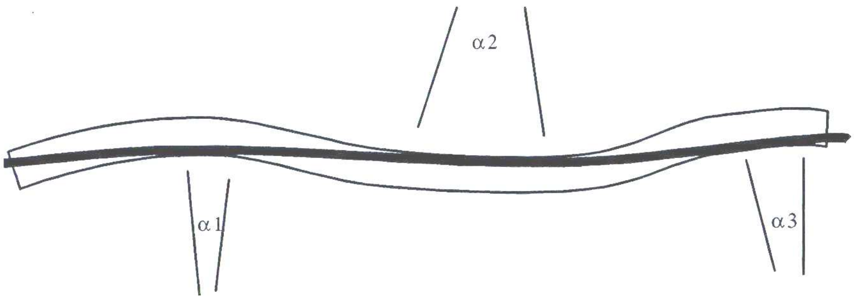


Figure 6.4: True path of the duct

The tendon touches by friction the duct in every parasite contact.

k is the additional arithmetic angular deviations by duct length unity.

$$k = \frac{|\alpha 1| + |\alpha 2| + \ldots + |\alpha n|}{l}$$

The tension P in the straight part of the tendon, calculated at a length x from the origin is written:

$$P = Po \times e^{-\mu k x}$$

Even in the curve part, the tendon trace can change from the hypothesis trace, because of the imperfection of construction. In this area, it exist some additional parasites deviation and friction.

The angular deviation α is determinate by the trace of the cable defined by the projector. The parasite deviation represented by k is unpredictable and dependent essentially of the nature and the size of the reinforcement and the care gives at the execution, and the intensity of the vibration of concrete.

The tension P in any point of the cable represents the function the tension at the anchorage by the formula:

$$P = Po \times e^{-\mu(\alpha + kx)}$$

Nota :X being the abscise of the section, where p is calculated. In theory, it is the curvilinear abscise of the tendon. In practise, we take in consideration the abscise in the length of the longitudinal axe of the beam.

A cable formed with strands has for value:

μ = 0.16 to 0.20 rd^{-1} and k = 0.005 to 0.01 rd/m^{-1}

Usually, $\mu \times (\alpha + kx)$ is small; the tension variation in the cable is a function quasi-linear to the abscise.

Estimate during the design of the project, the values of μ and k can be controlled during the execution. It is in fact, a global verification at the beam scale. During the tensioning, the measures are for the cable strain, elongation, in function of the force applied to the jack and also of the residual force at the passive extremity of the cable (measure the transmission coefficient) in one side. With enough different cables traces, the measure of transmission coefficient allow to determinate acceptable values, in the static way of parameters μ and k.

Tension of a tendon in its two extremity

If a tendon is under tension in its both extremity, it exist a point in the central zone, which is not subject to any displacement. In this area, the tensions are balancing each other. In the common case the tendon trace is symmetric, the point is located at the half of the length. In diverse case, it is located at the abscise such as the loss at the left would be equal to the loss at the right (the forces at the jack are supposed similar).

Example:

In this example, I follow the process of Patrick Le Delliou[1], which used the Eurocodes to find the losses, this example will carry on with the loss by anchorage draw-in and by the losses by creep, shrinkage and relaxation phenomenon. The example has different numerical data but takes the same hypothesis.

Consider a tendon trace is defined by two lean straight segment of 1 m just after the anchorage, and two parabolic segment connected to one horizontal straight segment near to the bottom fibre of the beam.

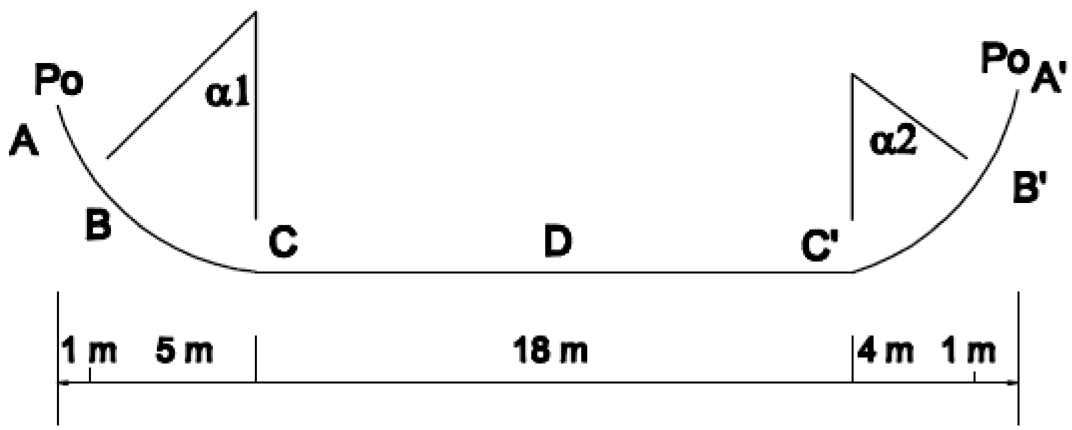


Figure 6.5: Cable geometry

We try to determinate the values of the tension all a long of the tendon with the following hypotheses:

Po = 700 kN $\alpha 1 = 0.52$ rd

μ = 0.18 rd^{-1} $\alpha 2$ = 0.50 rd

k = 0.008 rd/m

Tension force in B (xb=1)

$$P_B = Po \times e^{-\mu k\, xb} = 699.00 \text{ kN}$$

Tension force in C (xc=6)

$$P_C = Po \times e^{-\mu(\alpha 1 + k\, xc)} = 631.97 \text{ kN}$$

Tension force in B' (xb'=1 from the right anchorage)

$$P_{B'} = P_B = 699.00 \text{ kN}$$

Tension force in C' (xc'=6 from the right anchorage)

$$P_{C'} = Po \times e^{-\mu(\alpha 2 + k\, xc')} = 635.16 \text{ kN}$$

Fixed position D

In D, it is:

$$P_D = Po \times e^{-\mu(\alpha 1 + k\, xd)} = Po \times e^{-\mu(\alpha 2 + k\,(1\text{-}xd))}$$

so,

$$\alpha 1 + k \times xd = \alpha 2 + k \times (1 - xd)$$

$$xd = \frac{(k \times l - \alpha 1 + \alpha 2)}{2 \times k} = \frac{(0.008 \times 29 - 0.52 + 0.50)}{2 \times 0.008}$$

$$xd = 13.25m$$

$$P_D = 625.40\,kN$$

The diagrams the tension after friction loses looks like figure 6.6. The minimum is located in fixed point D. The tension in both extremity is by hypo these equal to the initial tension in the jack.

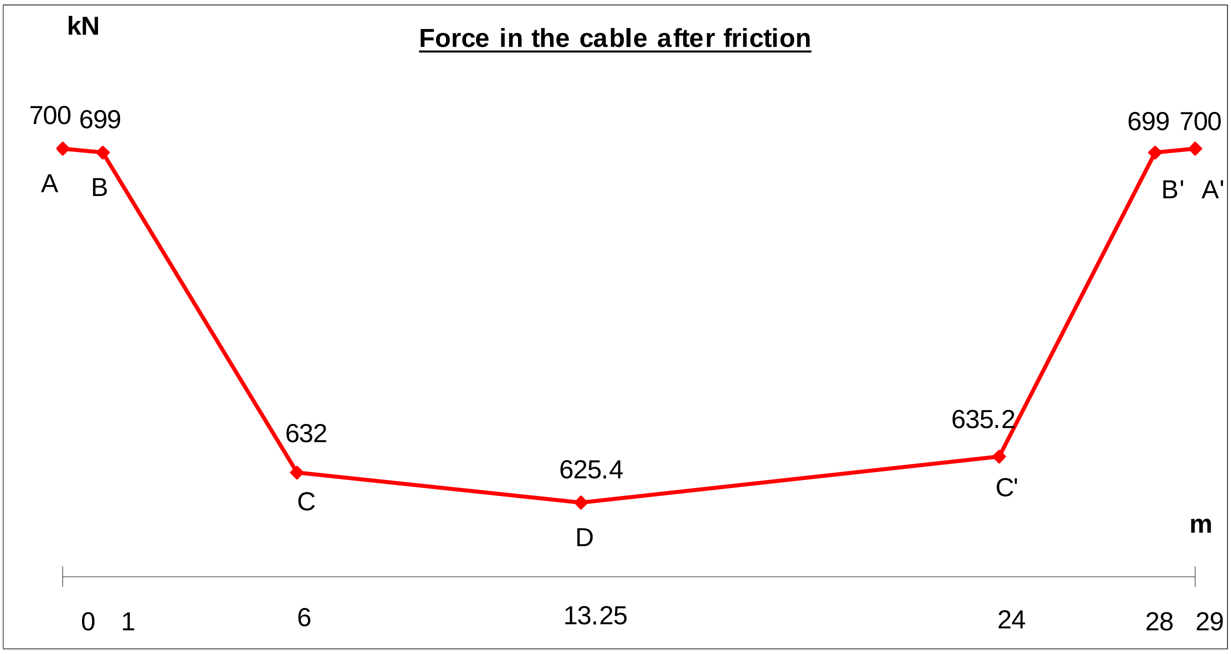


Figure 6.6: Force in the cable after friction

Anchorage draw-in:

Anchorage draw-in is the mean from the jack to the anchorage. This is during the process of transferring the tensioning force a little contraction feel by the prestressing tendons. The kind of anchorage does depend on the exact quantity of contraction. For the pretensioning the contraction may be waged by initially over-extending the

tendons by the calculated quantity of anchorage draw-in. This effect is much greater on a short prestressed concrete member than on a long one. For the post tensioning the effect is reduced by the friction of that exists between the tendons and the ducts because the tendons return due to the draw-in.
Many anchorage systems can use wedges to grip the tendon and transfer the tendon force to a solid steel anchorage poor in the concrete. The solid anchorage has some small deformation and most of the contraction in the length of the tendon takes place as a result of slip between the tendon and the wedges. For this system the typical value is 5mm.

Actually during the force transfer from the jack to the anchorage, there are small movements from the tendon to the concrete. These movements can be inexistent or neglected by using certain anchorage but for the conic anchorage the movement is important. The displacement g of the tendon to the interior of the duct is disturb by the friction forces. At a distance X, the friction forces sum balance the tension loss due to tendon slide and there are not anymore the displacement of the prestressing steel in the duct. The squeeze of anchorage do not product loss of tension after the section located at the abscise x.

The friction duct/tendon is symmetric: There is the same loss of friction when the tendon moves right to left or left to right.

If it is admitted that the variations of prestress are linear in x, the stresses diagram in the steel, before and after anchorage draw in can be presented by the symmetric line as the parallel of the axe x. It is possible also to put in ordinal axe not the tension but the force in the tendon or in the tendons by multiplied the stress by steel section appropriate.

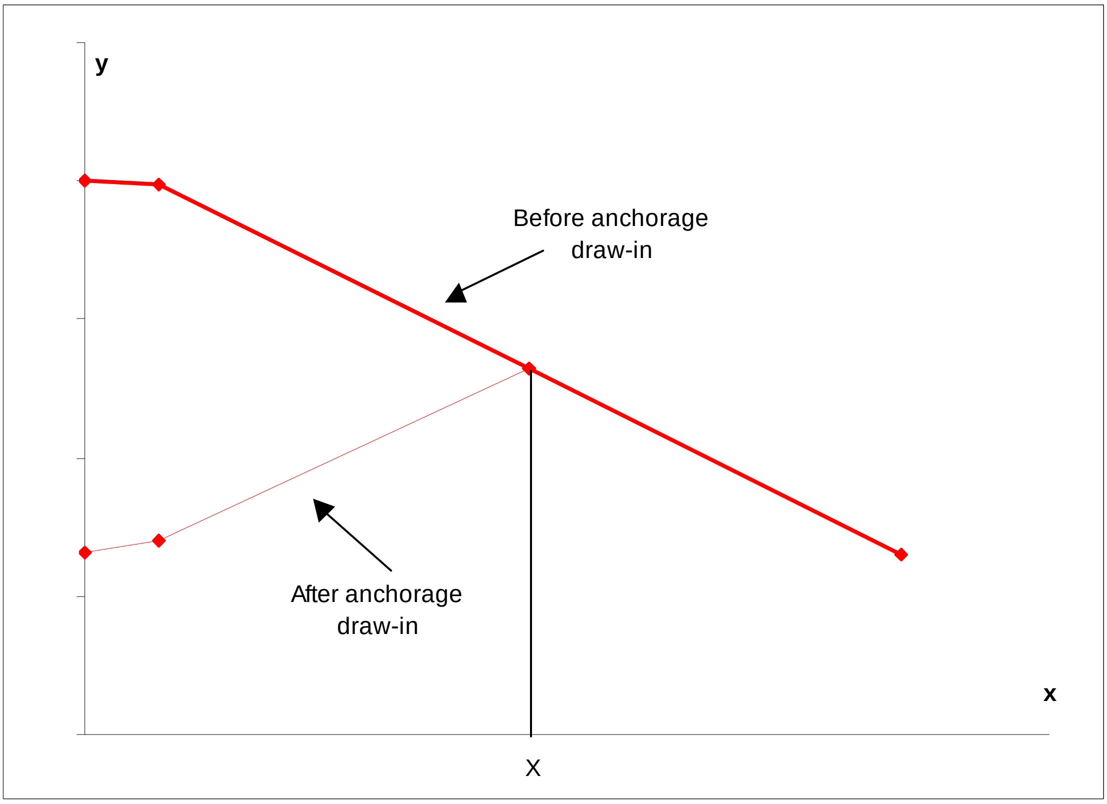


Figure 6.7: Anchorage draw-in effect

In the absence of hypothesis on the linear curve formed, it will be easy to manipulate the hyperbolic function but this will make very harder and heavier the calculation for a small finale improvement result.

During the anchorage draw in, a segment of the tendon with a length dx get shorter of δdx. It is admitted that the steel behaviour is elastic and follows the Hook law. The prestress loss in the steel, because of the draw in has for value:

$$\frac{\delta P}{A_p} = E_p \times \frac{\delta(dx)}{dx}$$

$$\delta(dx) = dx \times \frac{\delta p}{A_p E_p}$$

Ap is the tendon section and δp is the difference between the force before and after the anchorage draw-in.

Then, it is evidently:

$$g = \int_0^x \delta \ (dx)$$

So: $g = \dfrac{1}{A_p E_p} \times \int_0^x \delta P \times dx \rightarrow g\, A_p\, E_p = \int_0^x \delta P \times dx$

This integration represents the area between the two forces diagrams, before and after anchorage draw-in.

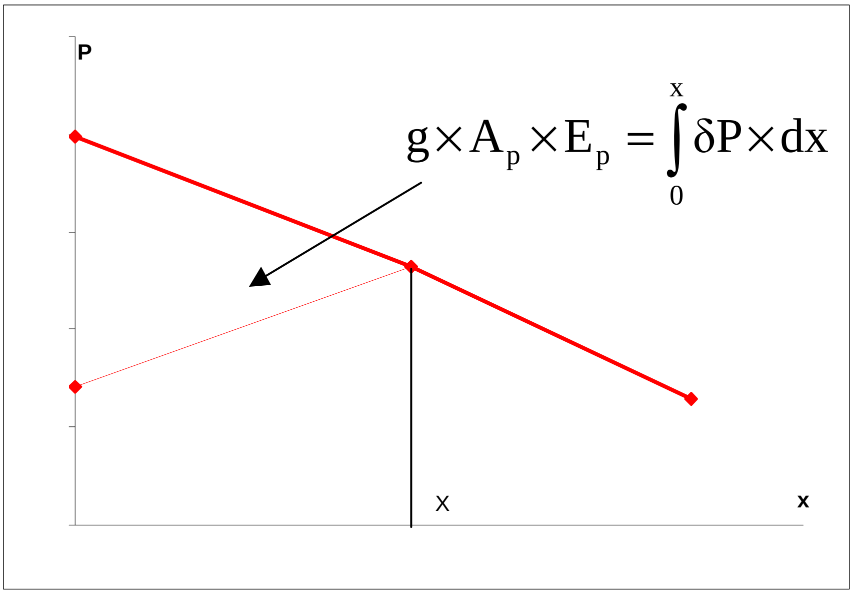


Figure 6.8: Integration representing the area before and after anchorage draw-in

The calculation of X and the tendon tension occurs when:

- the diagrams are symmetrical as the horizontal axe;
- the area between this diagrams are equal to gApEp

Generally, the tendon trace is composed of straight segments and curve segments. The linear tension diagrams before the anchorage blockage is constituted of variable slope. To do the loss calculation by anchorage draw-in. It must know in what area is the X abscise, which in the figure 6.9 can be between A and B, or B and C, or after C.

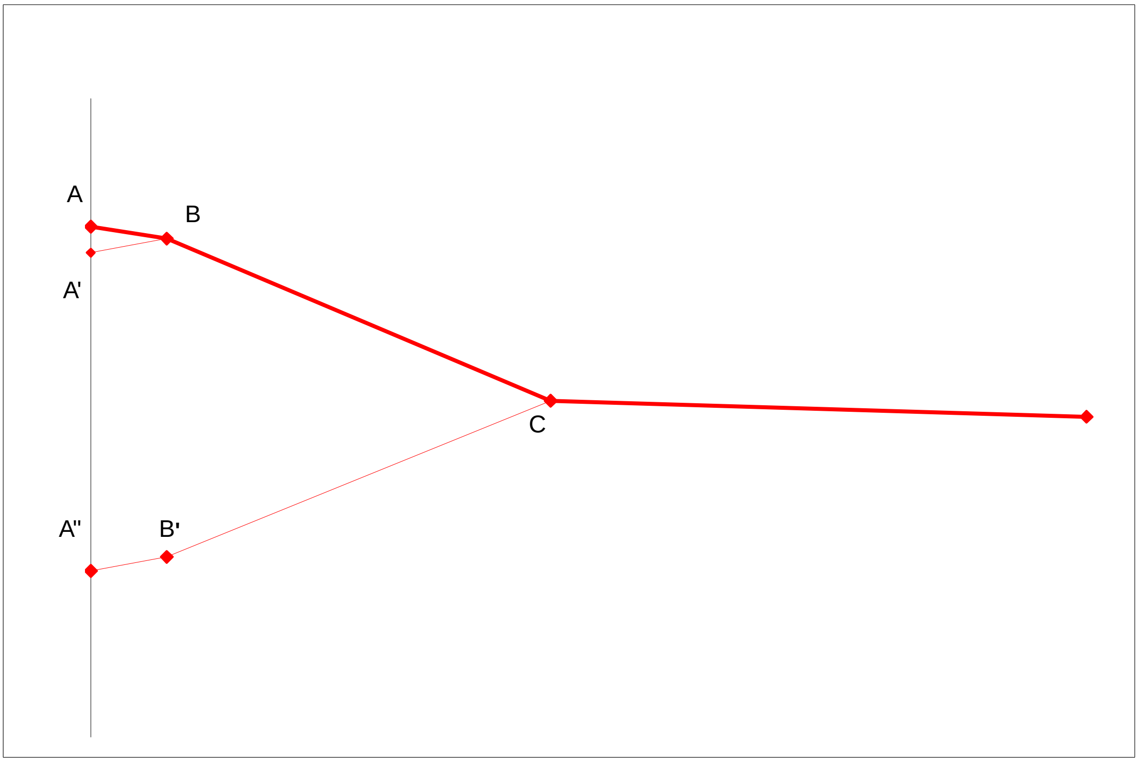


Figure 6.9: Part interested by the draw-in

The simple solution will consist to compare gApEp with the surfaces ABA', ABCB'A"… In this way, it is possible to determine the segment where is located X.

For example, if X is between B and C, it is:

Surface ABA' < gApEp < Surface ABCB'A"

Example:

The cable studied previously has a section equal to 462 mm² (cable 12ϕ7)and it is subject to an anchorage draw-in to 5mm.

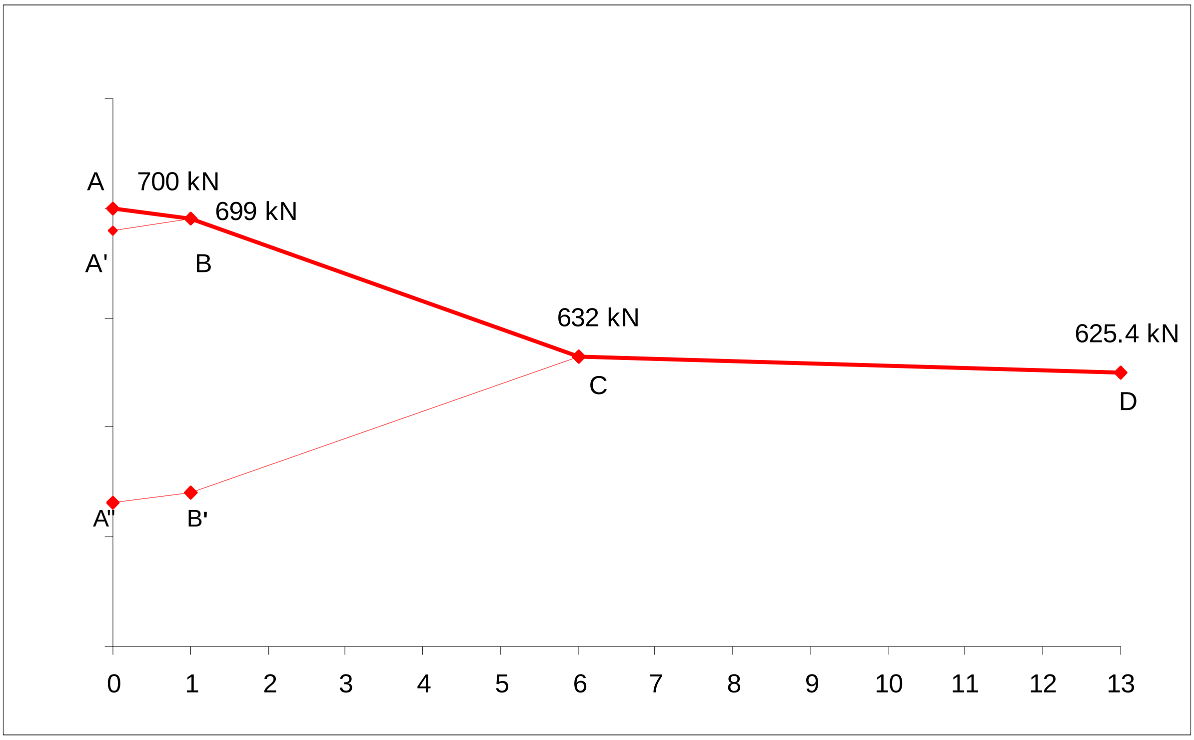


Figure 6.10: Calculation of partial area

g = 0.005 m

A_p = 462 mm

E_p = 200000 Mpa (value from the manufacturer)

Determination of the area containing the point X:

Area ABA' = (700-699)×1m = 1 kNm

$1 < g\ A_p\ E_p = 0.005 \times 462 \times 10^{-6} \times 200000 = 462$ kNm

The area ABA' is smaller to g A_p E_p. So X is located after the point B.

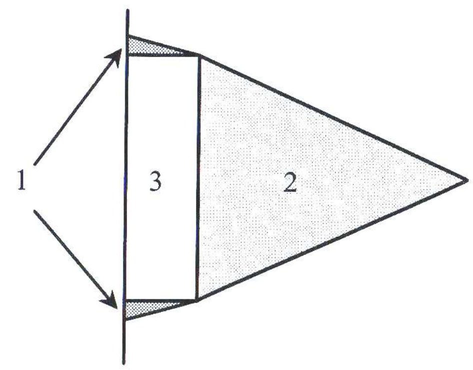


Figure 6.11: Calculation of the area ABCB'A"

Area ABCB'A" = (1) + (2) + (3)

(1) = (700 - 699)×1 = 1.0 kNm

(2) = (699 - 632)×5 = 335.0 kNm

(3) = 2×(699 - 632) = 134.0 kNm

(1) + (2) + (3) =470 kNm > g A_p E_p. (462 kNm)

So X is located between B and C

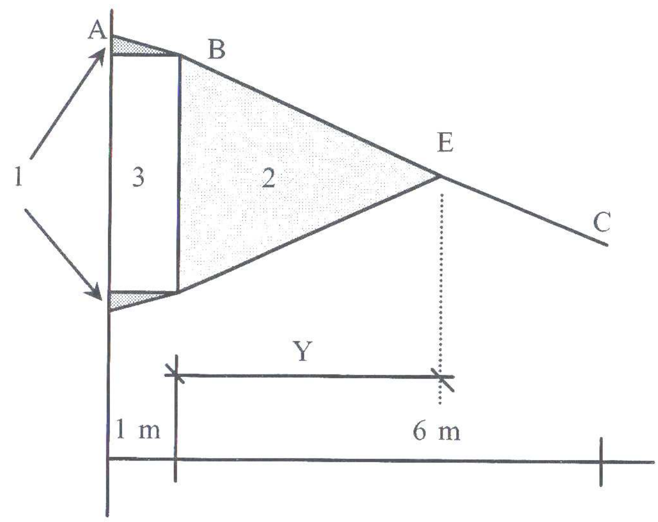


Figure 6.12: Draw-in between B and C

For the calculation of X, it is taken: Y = X – 1 (Y is the distance from the point E from BB').

It can be written:

$$\frac{P_E - P_C}{P_B - P_C} = \frac{5 - Y}{5}$$

$$P_E = P_B - \frac{Y}{5} \times (P_B - P_C)$$

$$P_C = 632\text{kN} \qquad\qquad P_B = 699\text{kN}$$

$$P_E = 699 - \frac{Y}{5} \times (699 - 632)$$

$$P_E = 699 - 13.4 \times Y$$

The area (1), (2) and (3) are for value (in kN)

(1) $= P_A - P_B = 1$

(2) $= (P_B - P_E) \times Y = 13.4 \times Y^2$

(3) $= 2 \times (P_B - P_E) = 26.8 \times Y$

If by equalling the total area to g A_p E_p.:

$462 = 1 + 26.8\ Y + 13.4\ Y^2$

This second-degree polynomial equation has only one positive result, so Y = 4.95m. It is possible now to calculate P_E, $P_{A'}$, $P_{B'}$:

$$P_E = 699 - 13.4 \times 4.95 = 632.67\ \text{kN}$$

$$P_{B'} = P_B - 2\times(P_B - P_E) = 566.34\,\text{kN}$$

$$P_{A'} = P_{B'} - (P_A - P_B) = 565.34\,\text{kN}$$

Tension of a tendon in its two extremity

Sometimes the draw-in can happen after the fix point of the cable in the case of tensioning by the both extremity, middle if the cable trace is symmetric. This phenomenon happens when the friction is small and the anchorage draw-in is important such as conic anchorage and traversals cables, which ensure the prestress for some bridge. In this hypotheses, it is better to tight the cable by only one extremity.

Example:

A symmetric cable of 18 m, which looks like figure 6.13. The cable has for section Ap =462 mm², it is under a prestress force P_A to the anchorage of 700 kN and has the followings parameters:

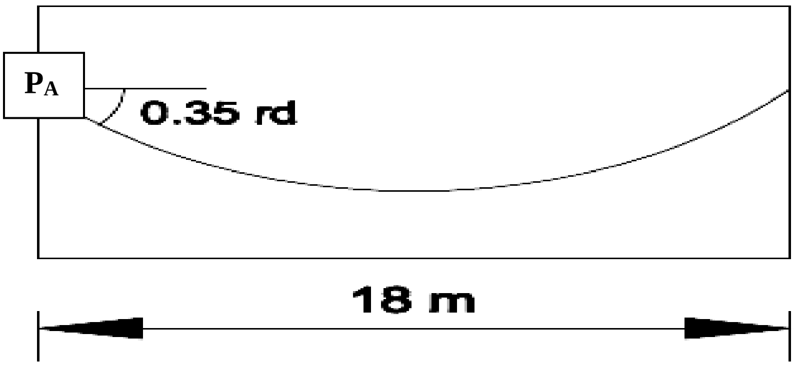


Figure 6.13: Cable section

P_A = 700 kN g = 8 mm

μ = 0.18 rd^{-1} k = 0.008 rd/m

- Cable tensioning from the both extremity

The cable is symmetric, the fix point C is at the middle (x = 9 m).

So,

$$P_{(x)} = P_A \times e^{-\mu(\alpha(x)+k\,(x))}$$

For x = 9 m and α = 0.35 rd, it is:

$$P_c = 700 \times e^{-0.18(0.35+9\times0.008)} = 648.8\ \text{kN}$$

Before the anchorage draw-in and supposing the linear variation between the anchorage, the diagrams of tensions is as figure 6.14:

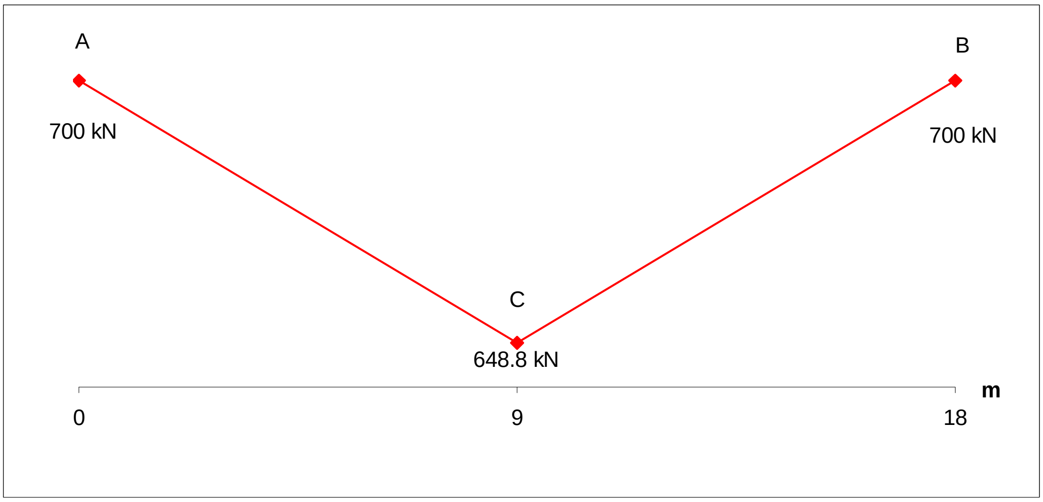


Figure 6.14: Linear strength in the cable

The cable being tensioned in both extremity, the anchorage draw-in cannot be possible after the point C. Two cases may happened:

The anchorage draw-in in A does not have repercussion after the point C

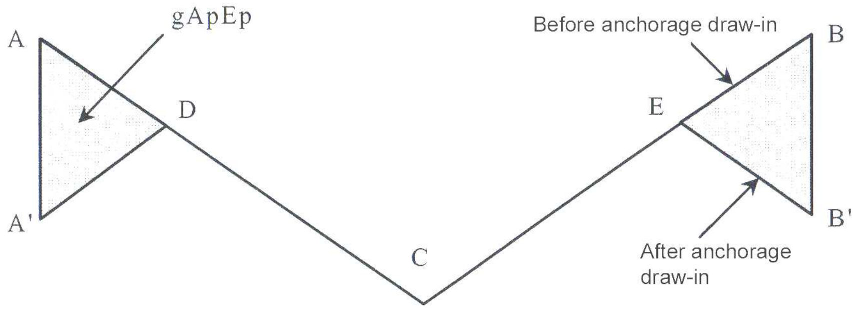


Figure 6.15: Effect of a small anchorage draw-in

The anchorage draw-in has a repercussion after C, it is impossible here because it is a symmetric beam.

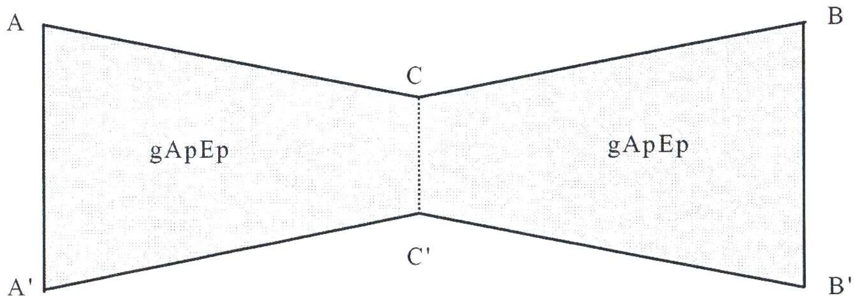


Figure 6.16: Effect of an important anchorage draw-in

To determine the hypotheses to keep, it is possible to compare the value gApEp to the surface of the diagrams limits, where C and C' are mix up.

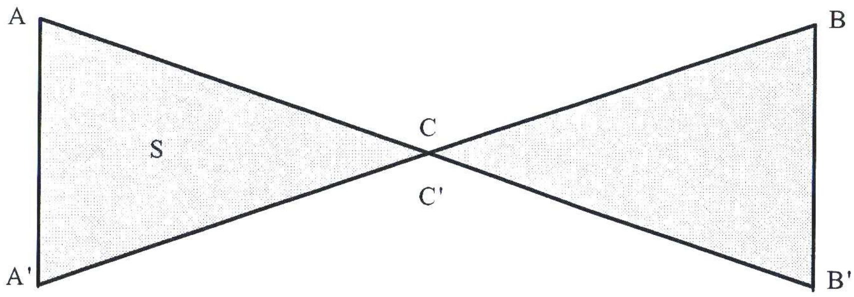


Figure 6.17: Anchorage draw-in reaching the middle of the beam

In this configuration, the calculation is:

$$S = 9\times(P_A - P_C) = 9\times(700 - 648.8) = 460.8\,\text{kN.m}$$

but gApEp = $0.008 \times 462 \times 10^{-6} \times 2 \times 10^{8} = 739.2$ kN.m > S

The tension diagrams, after anchorage draw-in is like figure 6.16. The unknown is not the fix point but the additional loss P_c-$P_{c'}$.

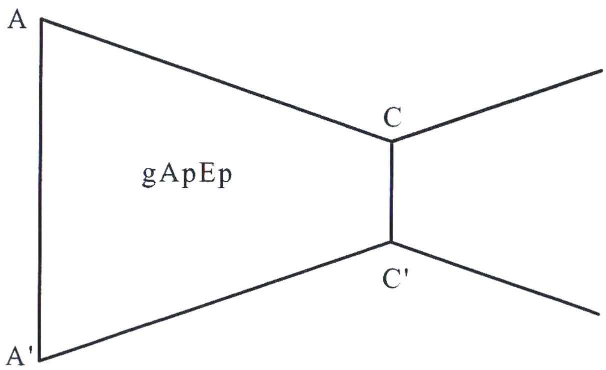


Figure 6.18: Calculation of an important anchorage draw-in effect

GApEp = Area ACC'A'

$739.2 = (P_A - P_C) \times 9 + (P_C - P_{C'}) \times 9$

where;

$$P_C - P_{C'} = \frac{739.2 - 9\times(700 - 648.8)}{9} = 30.9\,\text{kN}$$

$P_{A'} = P_{B'} = 700 - 2 \times (700 - 648.8) - 30.9 = 566.7$ kN

$P_{C'} = 648.8 - 30.9 = 617.9$ kN

- Cable tensioning from only one extremity

The anchorage draw-in has a repercussion until the point D located at the abscise X such as:

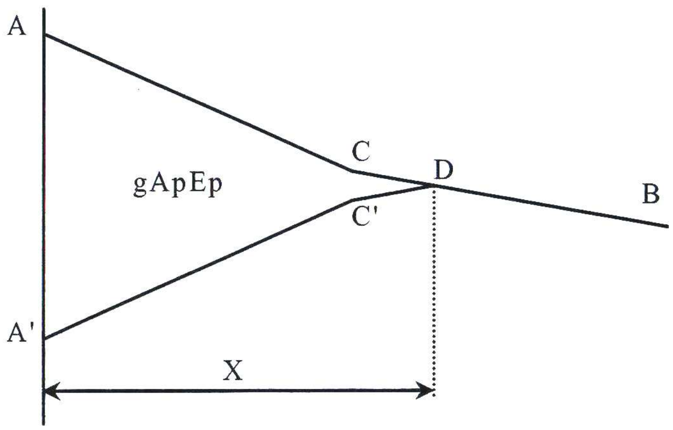


Figure 6.19: significant draw-in for a cable tensioned in one extremity

$$P_B = 700 \times e^{-\mu(\alpha + k \times xB)}$$

$$P_B = 700 \times e^{-0.18(0.35+18\times 0.008)} = 640.44 \text{ kN}$$

$$P_D = 648.8 - \frac{Y}{9} \times (648.8 - 640.44)$$

$$P_D = 648.8 - 0.93Y$$

The area (1), (2) and (3) are for value (in kN)

(1) = $X_c \times (P_A - P_C) = 9 \times (700 - 648.8) = 460.80$ kN

(2) = $2 \times X_c \times (P_C - P_D) = 2 \times 9 \times (648.8 - 648.8 - 0.93\,Y) = 16.74\,Y$

(3) = $(Y - XC) \times (P_C - P_D) = (Y - 9) \times (648.8 - 648.8 - 0.93Y) = 0.93Y^2 - 8.37Y$

If by equalling the total area to g A_p E_p.:

$739.2 = 460.80 + 8.37\,Y + 0.93\,Y^2$

This second-degree polynomial equation has only one positive result, so x = 13.38m. It is possible now to calculate P_D, $P_{A'}$, $P_{B'}$, $P_{C'}$:

$$P_D = 636.35\,\text{kN}$$

$$P_{A'} = -700 + 2 \times 636.35 = 572.71\,\text{kN}$$

$$P_{C'} = 2 \times P_D - P_C = 623.90\,\text{kN}$$

$$P_{B'} = 700 \times e^{-0.18(0.7 + 18 \times 0.008)} = 601.3\,\text{kN}$$

This tension is bigger as the tension obtained the cable in both extremity.

If a prestressed beam is stress by two similar cables, the figure 6.20 gives the values of the final tensions in the beam. When the two cables have tension by both extremities and when the two cables are tensioned once by the left extremity and once by the right extremity.

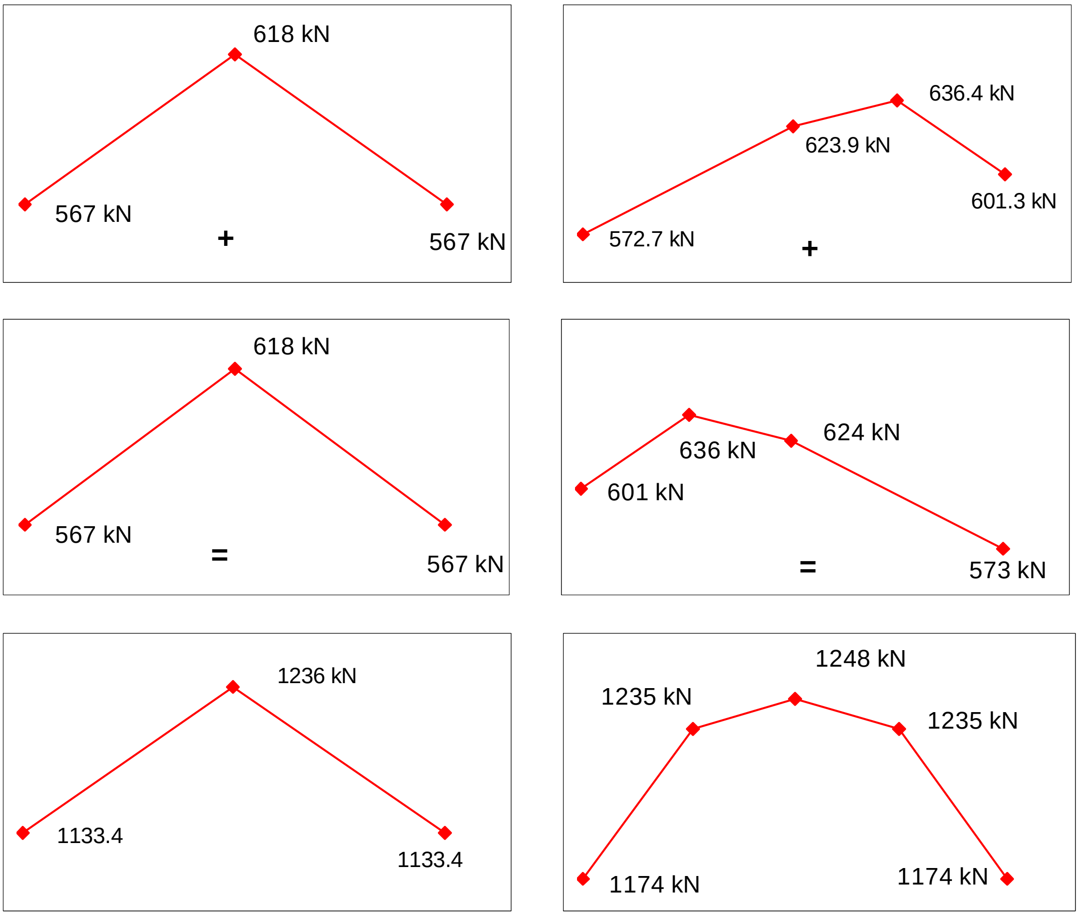


Figure 6.20: Comparison of tensioning by one or two extremities

The bigger value on the right part of the figure 6.20 illustrates that it is better to tensioned by only one extremity because the loss of prestress

Non-simultaneity of the tensioning:

Some engineers consider one more short loss of prestress during the tensioning if there are several tendons, they are not tensioned at the same time. This problem is a practical matter from three reasons:

- economically, the simultaneous tensioning required a lots of jack in the site;

- practically, the space of the jack will be so large, this will necessitate distance between anchorage axe very important;
- technically, for the structure construct by phases, this involves successive tensioning on the members.

The fact to not tension the tendons of the same beam produce loss of prestress. Indeed, after the first tendon been tensioned and been in the anchorage, the tensioning of the second tendon provokes strain in concrete. The first tendon tensioned is subject to the same deformation and this incite tension reduction. If the beam has n tendons, the first is subject to n – 1 tendon effect following and so on. Only the last tendon does not have this effect.

The exact calculation of the prestress loss produce from the non-simultaneity of tensioning is in theory complex. It must calculate the concrete shorting strain in the tendon 1 at j when the tendon J+1 is tensioning, and after the tendon j and j+1 when the tendon j+2 is tensioning, etcetera. In some special case, the tensioning of a tendon can provoke the elongation to an another tendon tensioned then there are not loss but adds of tension.

Patrick Le Delliou[1] explain the phenomenon by considering a beam, with a transversal section Ac and an inertia I, prestressed by similar n tendons. Take the hypotheses that the tendons have the same prestress force P and have he same trace with an eccentricity of eo.

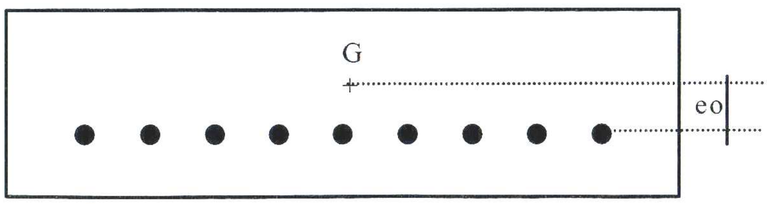


Figure 6.21: Prestress by several cables

Supposing in the first time we are neglected the self-weight. In the tendons gravity centre, each tensioning occur the stress $\delta\sigma_c$ on the concrete.

$$\delta\sigma_c = \frac{P}{A_c} + \frac{Peo^2}{I}$$

The relative concrete shortening supposed to follow Hook law has for value:

$$\frac{\delta l}{l} = \varepsilon_c = \frac{\delta\sigma_c}{E_c}$$

nota: E_c being the instantaneous concrete moduli

The cables already tensioned are subject to the same relative concrete shortening $\varepsilon_p = -\varepsilon_b$ and consequently there is losses of stresses:

$$\delta\sigma_p = \varepsilon_p \times E_p = -\delta\sigma_c \times \frac{E_p}{E_c}$$

The first cable tensioned is subjected to a tension variation because n-1 cables followings. Stress speaking, the total loss in the first cable will be:

$$\delta\sigma_{p1} = -(n-1) \times \delta\sigma_c \times \frac{E_p}{E_c}$$

The second cable received the effect from the (n-2) followings, equal to:

$$\delta\sigma_{p2} = -(n-2) \times \delta\sigma_c \times \frac{E_p}{E_c}$$

After tensioned all the cables, it is:

$$\delta\sigma_{p1} = -(n-1)\times\delta\sigma_c \times \frac{E_p}{E_c}$$

$$\delta\sigma_{p2} = -(n-2)\times\delta\sigma_c \times \frac{E_p}{E_c}$$

$$\delta\sigma_{p3} = -(n-3)\times\delta\sigma_c \times \frac{E_p}{E_c}$$

……………………………………

$$\delta\sigma_{pj} = -(n-j)\times\delta\sigma_c \times \frac{E_p}{E_c}$$

……………………………………

$$\delta\sigma_{p(n-1)} = \delta\sigma_c \times \frac{E_p}{E_c}$$

$$\delta\sigma_{pn} = 0$$

If A_p is the cable section, the total loss δ_p from prestressed force for all the cables is written:

$$\delta P = A_p \sum_{j=1}^{n} \delta\sigma_{pj} = -\frac{n(n-1)}{2} \times A_p \times \delta\sigma_c \times \frac{E_p}{E_c}$$

Hence $n \times \delta\sigma_c$ represents the concrete stress where the average cable pass, if it will be tensioned the n cables in the same time.

In average, the value of the loss of tension in a cable is:

$$\delta\sigma_p = -\frac{\sigma_c}{2} \times \frac{n-1}{n} \times \frac{E_p}{E_c}$$

Where $\sigma_c = n \times \delta\sigma_c$ correspond to the stress in the concrete after the tensioning of the n cables supposed simultaneous

Permanent load influences:

The precedent calculation is possible in space. But in reality, the self-weight act on the beam during the tensioning. The exact calculation of this phenomenon is complex because it depends of the perfect time when the weight is taking into account and by consequence incites the structure to be more rigid during tensioning:

- If the hanger is noticeably rigid, the beam raises itself during the tensioning of the first cable. The concrete weight has no importance on the tension of the cable (the jack apply a tension and not an elongation) al well for the others cables.
- If the hanger is noticeably floppy, the concrete is not detach from the hanger during the end of tensioning. When the hanger is going away this active the concrete weight which induce effects on the totality of the cables.

σg is the concrete stress when the concrete is close to the tendon due to the permanent loads.

$$\sigma_g = \frac{Mg \times e_o}{I}$$

The deformation in concrete due to the weigh and is by consequence the deformation of the cables, is:

$$\delta_{\varepsilon p} = -\delta_{\varepsilon b} = -\frac{\sigma g}{E_c}$$

$$\delta\sigma_p = -\sigma g \times \frac{E_p}{E_c}$$

If only a full calculation using theory and in the hypotheses it is known the exact value of the tensioning, It is here admitted that the permanent loads effect applied at the tensioning is between this two extremes cases. By consequences, the tension loss in a cable is equal to:

$$\delta\sigma_p = -\frac{\sigma g}{2} \times \frac{E_p}{E_c}$$

σg is the stress in the concrete under the effect of the permanent loads existing at his instant.

In most of the case, the permanent load is modified after the tensioning. The variation matching the force applies itself in whole cables.

The prestress variation from the tensioning in several phases is written:

$$\delta\sigma_p = -\frac{E_p}{E_c} \times \left[\frac{n-1}{2n} \sigma_c + \frac{\sigma_{cp1}}{2} + \sigma_{cp2} \right]$$

σ_{cp1} represent the concrete stress beside the average cable under the effect of the permanent loads added during the tensioning, and σ_{cp2} is equal to σ_{cp1} with added the permanent force after the tensioning.

In a practical formulation usually (n-1)/n is replace by 1. This formulation is definitely an approximation and this means the evaluation of the loss are by excess. It is useful only if the cables are very close to each other because the concrete stress beside each can be neglected. And in reality this approximation has not a huge impact because δ_{cp} is always small.

Example

Considering the cable already calculated for the friction and the anchorage draw-in. After these losses, the tension diagrams in the left part of the cable is equal to:

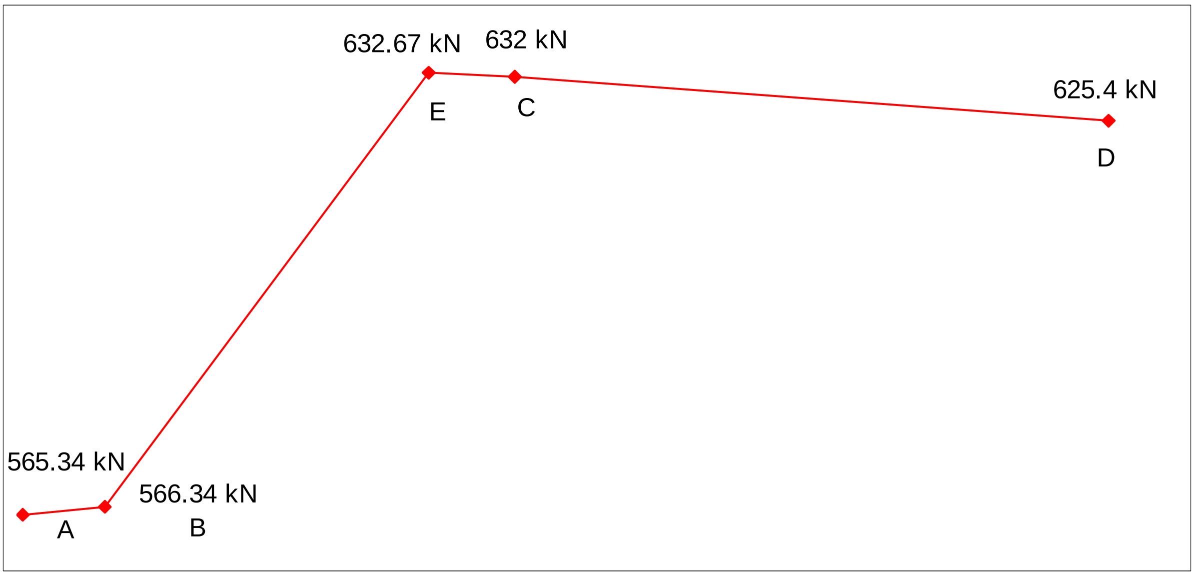


Figure 6.22: Force in the cable after friction and anchorage draw-in

Considering that it is the middle cable equivalent to several armature tensioning successively and that the stresses in the concrete beside the cable are below. And these stresses are under effect of all cables tensioned in the same time and from the self-weight.

$\sigma c_A = \sigma c_B = 5$ Mpa

$\sigma c_C = \sigma c_E = 7$ Mpa

$\sigma c_D = 9$ Mpa

$E_p = 200000$ Mpa

$E_c = 35000$ Mpa

There are no permanent loads applied after the tensioning. It is deduct the followings losses:

$$\delta\sigma p_A = \delta\sigma p_B = -\frac{5}{2}\times\frac{200000}{35000} = -14.29 \text{ Mpa}$$

A_p =462 mm² so,

$$\delta p_A = \delta p_B = -14.29\times 462\times 10^{-3} = -6.60 \text{ kN}$$

$$\delta\sigma p_C = \delta\sigma p_E = -\frac{7}{2}\times\frac{200000}{35000} = -20 \text{ Mpa}$$

$$\delta p_C = \delta p_E = -9.24 \text{ kN}$$

$$\delta\sigma p_D = -\frac{9}{2}\times\frac{200000}{35000} = -25.71 \text{ Mpa}$$

$$\delta p_D = -11.88 \text{ kN}$$

The tension diagram in the cable, after all instantaneous losses is:

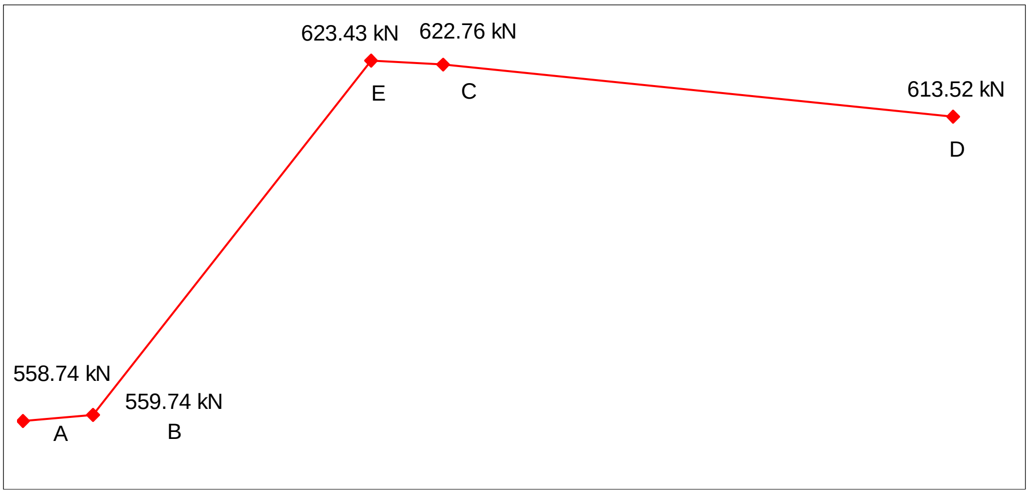


Figure 6.23: Force in the middle cable after none simultaneous tensioning

6.3.2- Long-term losses

Shrinkage

For the period of concrete hardening, the concrete is subject to a volume shrink called shrinkage (see part 3). The shrinkage is not finished when the prestressing tendons are tensioned. $\Delta\varepsilon s$ the shorten strain (positive) of a beam causing by the shrink which appears after the tensioning. The prestressing tendons, associate to the concrete, are subjected to the same strain, it is follow by a tension loss equal to :

$$\Delta\sigma_{pshrink} = E_p \times \Delta_{\varepsilon p} = -E_p \times \Delta_{\varepsilon s}$$

The shrinkage evolution with time has been the object of study in laboratory. Its value, at the instant t, depend of four major parameters:

- surrounding humidity;
- the ratio W/C in concrete;
- the percentage of steel associated to concrete;
- the sample thickness

The following graphic can represent the shrinkage evolution in function of the time.

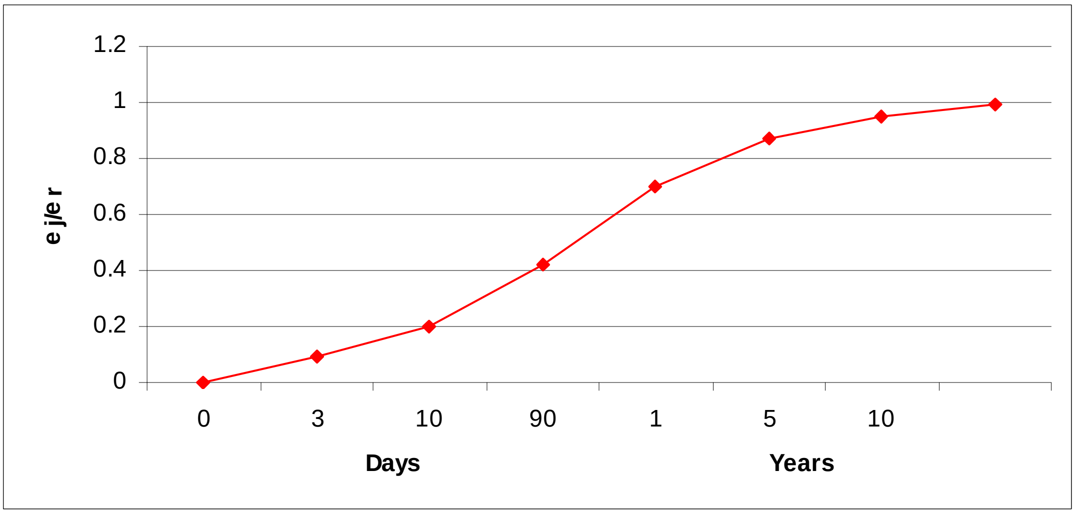


Figure 6.24: Shrinkage evolution

The curving line depends to two parameters, such as the aggregates (heavy, light, porous) properties and the concrete maintenance condition at young age and the size of the sample. Some techniques exist to reduce or accelerate the shrinkage.

When an analytic formulation is indispensable, it is possible to use the following equation for the sample.

$$\varepsilon r(j) = \varepsilon cd(\infty) \times \frac{j}{j + 0.04 \times ho^{1.5}} + \varepsilon ca(\infty) \times (1 - \exp(-0.2 \times \sqrt{j}))$$

J is in days, ho is the central radius of the sample. in millimetres (ho = 2Ac/U where Ac is the section and U the perimeter). εcd and εca are formulate is the section 3 (shrinkage).

The part from the endogen shrinkage (εca) rise rapidly its limit value. However it is not the case with the drying shrinkage, which can be very long. It is notice that the shrinkage between the pour of concrete and the prestressing do not any effect on the cables. In the formula giving the shrinkage loss, Δεr(t) is:

$\Delta\varepsilon r(t) = \varepsilon r(t) - \varepsilon r(to)$

(to) being the concrete age at the time of the prestress take place.

The loss of prestress by shrinkage is the same on the all beam, of course this hypotheses is not really true but the variation can be neglected.

Relaxation

A tendon tensioned at the stress σpi close to the elastic limit and with a length kept constant, result to the stress diminution with time.

This phenomenon is called relaxation, it depend to three major parameters:

- the steel chemical composition;
- the steel crystallography (from the thermals treatment)
- the tension in the steel

It is relatively easy, in laboratory to measure the steel relaxation. But it is different for a prestressed concrete member because the stress in the steel in approximately known and the stress is not constant in time (creep and shrinkage effect).

Generally, the relaxation is expressed in function of the stress σpi at the end of the tensioning, the relaxation ratio at 1000 hours.

The relaxation evolution in function of time can be obtain by the formula indicate in part 3. It is consider that the calculation at 500000h (about 57 years) represented the infinite relaxation ($t = \infty$). Henceforth we obtained the loss (negative) by relaxation Δσpr(t).

Creep

The concrete is not a perfect elastic ballast. Subject to a constant stress, the concrete is under deformation. Usually the deformation is twice in long-term to instantaneously.

Considering a prestressed beam by tendons with a force P(x), (at the end of tensioning) with an eccentricity e_o and a bending moment from permanent load M(x). The stress in the concrete, close to the tendon is:

$$\sigma_c = \frac{P(x)}{A_c} + \frac{P(x) \times e_o{}^2}{I} + \frac{M(x) \times e_o}{I}$$

Ec being the concrete, the concrete instantaneous deformation close to the cable is equal to.

$$\varepsilon_i = \frac{\sigma_c}{E_c}$$

In time, the concrete has a long-term deformation about the twice to the instantaneous deformation.

$\varepsilon_d = K \times \varepsilon_i \approx 2 \times \varepsilon_i$

The tendon is in theory after an infinite time, the same additional deformation as the concrete, equal to εi. The cable strain is like a tension loss $\delta\sigma_{pc} = -2 \times \varepsilon_i \times E_p$:

So,

$$\delta\sigma_{pc} = -2\frac{\sigma_c}{E_c} \times E_p$$

The formula from the section 3 allows a close evaluation of K.

Combination of the long-term effect:

Shrinkage of concrete, creep in the concrete and steel relaxation was discussed in the section of the materials properties, and they can be established separately and then summed. The Eurocode 2 states the following expression:

$$\Delta\sigma_{p,\,c+s+r} = \frac{\varepsilon_{sh} E_s + \Delta\sigma_{pr} + m\phi(\sigma_{cq} + \sigma_{cpo})}{1 + mA_p / A_c((1 + A_c e^2 / I_c)(1 + 0.8\phi))}$$

where:

- ε_{sh} is estimated shrinkage strain from Table 3.5;
- m is the modular ratio E_s/E_{cm} (where E_{cm} is based on the long-term value of concrete strength);
- ϕ is the creep coefficient from Table 3.4;
- $\Delta\sigma_{pr}$ is the variation of stress in the tendons due to steel relaxation(from Figure 3.4);
- σ_{cg} is the stress in the concrete adjacent to the tendons, due to self weight and any other permanent loads;
- σ_{cpo} is the initial stress in the concrete adjacent to the tendons due to prestress;
- e is the eccentricity of the tendons.

This equation is iterative because $\Delta\sigma_{pr}$ depends on the final value of prestress but the examination of Figure 3.4 shows that, with the usual initial stress in tendons of 0.7 f_{pk}.

For long-term losses of more than 15%, the relaxation losses are sensibly constant.

Table 6.2: Relaxation factors

	Class 1 (wire and strand)	Class 2 (wire and strand)	Class 3 (bars)
Pretensioning	1.5	1.2	-
Post-tensioning	2.0	1.5	2.0

Eurocode 2 giving this very complicate formula for the global loss of prestress hides the true numerical order of each phenomenon.

6.3.3- Total prestress losses

The total prestress losses is defined by the initial prestress force applied to a member Po and by the effective prestress force at transfer αPo and at design load βPo. The value α indicates the short-term losses due to elastic shortening, anchorage draw-in and friction, whilst the value β indicates the long-term losses due to concrete creep and shrinkage and steel relaxation.

During the initial design stage it is possible to approximate the prestress loss, which it will refined later in the design process when more details of the prestressing steel are available.

It is traditional to take 0.9 and 0.75 for the values of α and β for both pretensioned and post-tensioned members

Example:
In this example I follow the final steps of Patrick le Delliou[1] process to find the global loss of prestress. Studying the same cable from above, the tension in the cable at the end of the construction is equal to figure 6.22. The concrete shrinkage after tensioning has for value:

$$\Delta\varepsilon_r = 2\times10^{-4}$$

The stress in the concrete beside the cable is equal after instantaneously losses to:

$\sigma c_A = \sigma c_B = 4$ Mpa
$\sigma c_C = \sigma c_E = 6$ Mpa
$\sigma c_D = 8$ Mpa

These values are both ($\sigma cg + \sigma cpo$).

It is taking a cable steel of relaxation class 2. The cable characteristic resistance fpk is equal to 1660 Mpa.

Loss of prestress due to the shrinkage:

Whatever the section of prestress concrete the loss is wrote like this:

$$\delta\sigma p_r = -Ep\times\Delta\varepsilon r \qquad \delta P = -ApEp\times\Delta\varepsilon r$$

$$\delta P = -462\times10^{-6}\times200000\times2\times10^{-4} = -18.5 \text{ kN}$$

Loss of prestress due to the creep:

The loss in the section is given by:

$$\delta\sigma p_r = -2\sigma\sigma\times\frac{Ep}{Ec} \qquad \delta P = -2Ap\sigma A\times\frac{Ep}{Ec}$$

with Ec = 35000 Mpa, Ep/Ec = 5.71 then,

$$\delta P_A = \delta P_B = -2\times 462\times 10^{-6}\times 4\times 5.71 = -21.10\,\text{kN}$$

$$\delta P_C = \delta P_E = -2\times 462\times 10^{-6}\times 6\times 5.71 = -31.65\,\text{kN}$$

$$\delta P_D = -2\times 462\times 10^{-6}\times 8\times 5.71 = -42.20\,\text{kN}$$

Loss of prestress due to the relaxation

The stresses in the steel, after the only instantaneous losses are for value:

$$\sigma p_A = \frac{P_A}{Ap} = \frac{558;74\times 10^{-3}}{462\times 10^{-6}} = 1209.40\,\text{Mpa} \quad \Rightarrow \mu = \frac{1209.40}{1660} = 0.729$$

$$\sigma p_B = \frac{P_B}{Ap} = \frac{559;74\times 10^{-3}}{462\times 10^{-6}} = 1211.80\,\text{Mpa} \quad \Rightarrow \mu = \frac{1211.80}{1660} = 0.730$$

$$\sigma p_E = \frac{P_E}{Ap} = \frac{623.43\times 10^{-3}}{462\times 10^{-6}} = 1349.41\,\text{Mpa} \quad \Rightarrow \mu = \frac{1349.41}{1660} = 0.813$$

$$\sigma p_C = \frac{P_C}{Ap} = \frac{622.76\times 10^{-3}}{462\times 10^{-6}} = 1347.96\,\text{Mpa} \quad \Rightarrow \mu = \frac{1347.96}{1660} = 0.812$$

$$\sigma p_D = \frac{P_D}{Ap} = \frac{613.52\times 10^{-3}}{462\times 10^{-6}} = 1327.97 \text{ Mpa} \quad \Rightarrow \mu = \frac{1327.97}{1660} = 0.799$$

At the point A, and t infinite (equivalent to 500000 h), the loss by relaxation theoretically happening alone is equal to:

$$\Delta\sigma pr = \sigma pi \times k1 \times \rho 1000 \times e^{(k2\times\mu)} \times (\frac{t}{1000})^{0.75\times(1-\mu)} \times 10^{-3}$$

K1 and K2 can be found in the table in part 3.

$$\delta P_{Arel} = -\sigma pi \times A_p \times 0.66 \times 0.025 \times e^{(9.1\times 0.729)} \times (\frac{500000}{1000})^{0.75\times(1-0.729)} \times 10^{-3}$$

$$\delta P_{Arel} = -24.79 \text{ kN}$$

It will be the same way for the others, which have for results:

$$\delta P_{Brel} = -24.95 \text{ kN}$$

$$\delta P_{Erel} = -40.16 \text{ kN}$$

$$\delta P_{Crel} = -39.94 \text{ kN}$$

$$\delta P_{Drel} = -37.14 \text{ kN}$$

The long-term effect combination imposes more data. This calculation, effected to the point A with reasonable values of the different parameters, allow to obtain, $\Delta\sigma p(t = \infty) = -136$ MPa

The simple addition of the differed losses gives: - 64.39/0.462 = - 139 Mpa.

Finally, all losses considered; the prestress forces diagrams by cable are equal to Figure 6.25.

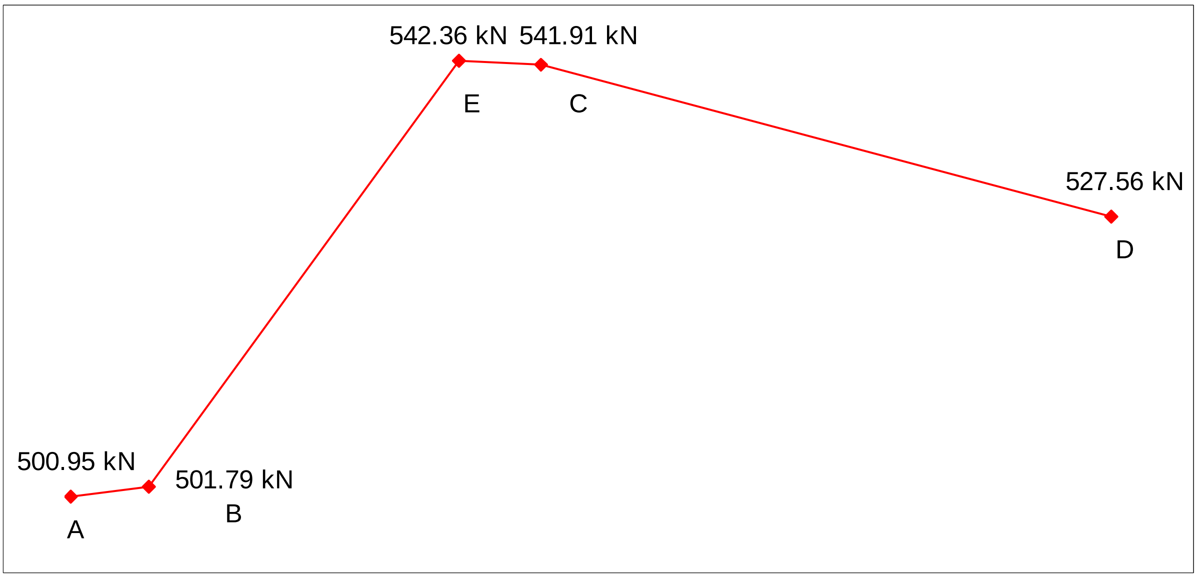


Figure 6.25: Force in the cable at t = ∞

Comparison of the losses:

The table 6.3 summarize the values of instantaneous losses and differed losses in different part of the section.

Table 6.3: Global prestress losses

	A	B	E	C	D
Instantaneous losses	134.66 kN	133.66 kN	67.33 kN	68 kN	74.6 kN
Long-term losses	64.39 kN	64.55 kN	90.31 kN	90.09 kN	97.84 kN
Total losses	199.05 kN	198.21 kN	157.64 kN	158.09 kN	172.44 kN
Percentage	28.4%	28.3%	22.5%	22.6%	24.6%

To give a numerical order, the total losses are in between 157 and 200 kN, and 22% to 29% of the force applied at the jack. This percentage may be reduced by improving the execution, especially the friction and the anchorage draw-in.

Part 7- Basis of design

7.1- INTRODUCTION

The design of prestressed members is usually ruled by limits on tensile and compressive stress in service and not by their strength at the ultimate limit state. Generally an preliminary design in the limitations on service stress is establish to choose an appropriate prestress force and tendon location. Afterwards the design is regarded to the ultimate limit state to check the strength requirements. If the bending is stronger than the strength of the member, ordinary reinforced bars might be given to enlarge the member moment capacity.

The prestressed concrete is designed under two critical loading conditions which occurs where concrete stresses must ensure the specified permissible values. The initial condition is known as the transfer condition. The concrete is at this point still moderately young and the concrete compressible strength has not reached its full design value. The transfer condition happens as soon as there is transfer of the prestress force to the concrete. At transfer condition the stresses are prestress and stress due to the moment, M_o, induced by the applied loads (which are present at transfer). However there is often only the self-weight of the member in that case the transfer moment is often equal to the self-weigh moment of the member.

The next condition is known as the service (SLS, Service Limit State) condition. The concrete is at this point mature and has full strength and the member is under a full service loads. At service condition the applied prestress force has been dropped off from its initial magnitude, P, due to losses in the concrete and the steel. The total loss of prestress force reaches 20 per cent between the transfer and the service condition.

The stresses taking into account the member in the service condition are the prestress and the stress from the moment, M_s, provoked by all permanent and variable loads.

7.2- CONCRETE PARAMETERS AFFECTING PRESTRESSED CONCRETE

Concrete is the main component of any prestressed concrete element. In prestressed concrete strength and endurance of concrete are the two principles virtues and must be achieved and ensure a strict quality control and quality assurance at any stage of the construction, from production to maintenance.

The hardening of concrete is classified into two categories according to its mechanical characteristics. The first mechanical properties of hardened concrete is the short-term properties, which are strength in compression, tension, and shear (stiffness as measured by the modulus elasticity). The second mechanical properties of hardened concrete is the long-term properties which are creep and shrinkage.

7.3- CONCRETE STRESS LIMIT

The prestress design must assure the limits on tensile and compressive stresses at transfer and service to guarantee the serviceability of the member. Using tensile stress limit is to make sure any cracking occur in the member. Also the utilization of the compressible stress limit is to limit microcraking and avoid extreme loss of prestress due to creep. In the Eurocode 2, the designer has a freedom to select the concrete stress limit according to his judgment on the appropriate case of study.
Moreover, the large deformations and loss of prestress from the creep of the concrete may be prevented. Eurocode 2 suggested that compressive stresses could not exceed 0.45 $f_{ck,0}$ at transfer and 0.45 f_{ck} at service. $f_{ck,0}$ and f_{ck} are the characteristic compressive cylinder strengths of the concrete at transfer and service, in that order.

In the occurrence of fully prestressed members, there is no cracking allowed. The tensile stress should be limited to the tensile strength of the concrete. According to the designers the tensile stresses in member will be prohibited at all in the service condition and evenly tolerate to rise during the transfer condition.

In the occurrence of partially prestressed members, there are cracks allowed to develop at service because stress limits are not generally precisely known at the service condition. Nevertheless limits are placed on the width to which cracks are allowed to open and for this motive the design of partially prestressed members is completed at the ultimate limit state and afterwards the transfer and service conditions are checked.

7.4- LIMIT STATE DESIGN

The engineer design structural element in limit state. The philosophy of limit state theory is to take into account different recognized conditions that affect the suitability of the structure. Each of these conditions have been defined from the structure situation, if one of the conditions are not fulfilled, the structure fails. The failure changes a lot between the limit states because they involve various factors of safety. There are two major groups of limit states for most structures. the ultimate limit state (ULS) usually means collapses and the serviceability limit state (SLS) generally known as deflection and cracking.

Ultimate limit state:

The ultimate limit state is reached when the structure or part of the structure collapses. The collapse can be from the loss of equilibrium, stability or to failure by rupture of structural members.

List of the three more important ultimate limit states:

- Strength. The structure has to be able to resist with a suitable factor of safety against collapse. Different collapses may be possible, as fracture of a single member, instability of the complete structure, or by buckling of section of the structure. If accidental overloading occurs an adequate factor of safety against collapse is required while the values are usually lower than that given for the service loads.

- Fire resistance. The structure should stand for sufficient time to let any occupants to run away.

- Fatigue. The structures subject to cyclic loading, especially important for prestressed concrete because the stress level in the prestressing steel is very high.

Serviceability limit state:

The serviceability limit state is reached when the structure while remaining safe but experience phenomena from basic daily used such as excessive deformation, cracking, vibration.

- Deflection. Under the service load, the deflection of the structure can not be excessive, if not damages occurs to finishes, partitions or cladding.

- Cracking. The cracking in the concrete can not only be unsightly but can conduct to extreme ingress of water into concrete, conducting corrosion of the steel.

- Durability. There is multiply risk of corrosion of the steel when the concrete is very permeable. The surrounding environment of the concrete may be aggressive, such as seawater. In the prestressed concrete the durability is induced by the concrete mix proportions, the cover around the steel and the grouting of post-tensioning ducts.

- Vibration. Typically used for structure such as machine foundation.

According to the factors of the structure, surrounding environment and the loads acting illustrated by the limit states, the structure should be identified and critically defined.

The two limit state designs have distinctive degrees of importance. This means that concrete elements which may have cracking will be affected by unsightly appearance and durability. However this problem is less serious than collapse of the whole structure due to the rupture of a critical member. To terminate this point, in limit state design, the factors used for the serviceability (SLS) are considerably inferior than factors used for safety (ULS), pointing up the better significance of preventing collapse of the structure.

Generally prestressed concrete structure design process firstly by regarding the serviceability limit state of cracking and afterwards verify the ultimate strength limit state.

7.5- CONCRETE BEAM UNDER PRESTRESS FORCE ONLY

Considering a beam in the longitudinal view with a constant section Ac, bent according to the radius R and prestressing by tendons in the neutral fibre with a force P. Because of the bending, the tendons strain press on the concrete and develop centripetal reaction, uniformly spread, with linear intensity of p = P/R.

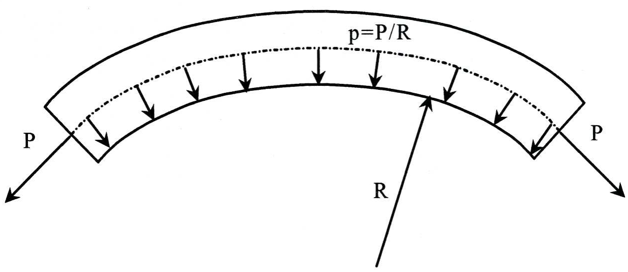


Figure 7.1: Beam under prestress force only

The beam is under a compression force produce by the prestress:

$$\sigma_c = \frac{P}{A_c}$$

During the bending effect, this force develops a centrifuge load; uniformly distribute which have for intensity by length unity of:

$$q = -A_c \times \sigma_c / R \quad = -P/R = -p$$

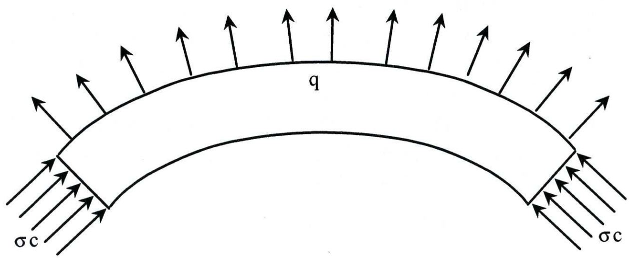


Figure 7.2: Stress in the beam

In fact, the compressive force from the concrete is equal to the tensile force of the tendons. A beam under uniquely the prestress force has only a longitudinal shorting deformation from the concrete compression.

Considering now the same beam in the transversal view.

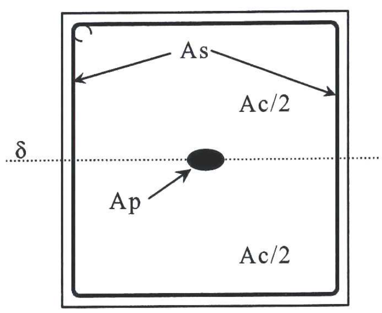


Figure 7.3: Transversal view

The tendons apply a force downwards. The concrete located under the pressure of the tendon apply a force upwards equal to q/2, the other half of concrete force being balance by the part located upper the tendons. The equilibrium in δ view show that there are a traction force equal to q/2 = -P/2.

There are a risk of cracking along of δ, to guarantee the balance in the system, it must disposed a link of section As by unity of length of beam:

$$A_s = \frac{p}{2 \times \sigma_{sadm}}$$

Generally P/2 is small and As is equal to zero. Usually this effect creates a stress in the vertical link.

Example: If P = 5.00 MN and R = 250m

P = 5/250 = 0.02 MN/M

A_s=0.02/ (2×333) =0.3 cm²/ml

But a special attention if the R is small

P = 5.00 MN and R = 12m

P = 5/12 = 0.416 MN/M

A_s=0.416/ (2×333) =6.25 cm²/ml

Considering the same beam but now with the tendon move from the centre to the bottom and we apply a moment M which reduce the stress in the inferior fibre.

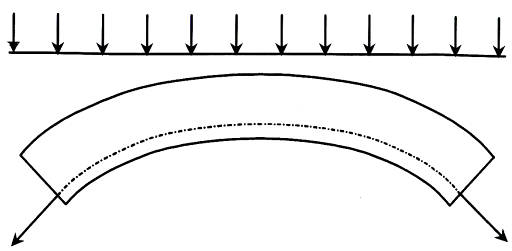


Figure 7.4: Beam with tendon in the bottom

Moving the tendon and adds the moment M do not change the total force on the section which still equal to P.

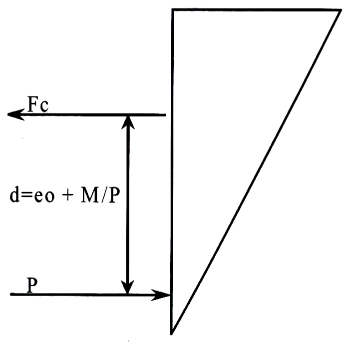


Figure 7.5: Cable eccentricity and stresses

But the total stress in the concrete and the prestress force are not coaxial. In figure 7.6, d is the distance between the tendons and the total stress. In this case the force is equal to:

In tendon: $p = P/R$ and concrete: $q = -P/(R+d)$

Anyway the equilibrium still remaining because the force in the length have inverse rapport:

$L1 = \alpha \times (R + d)$ and $L2 = \alpha \times R$

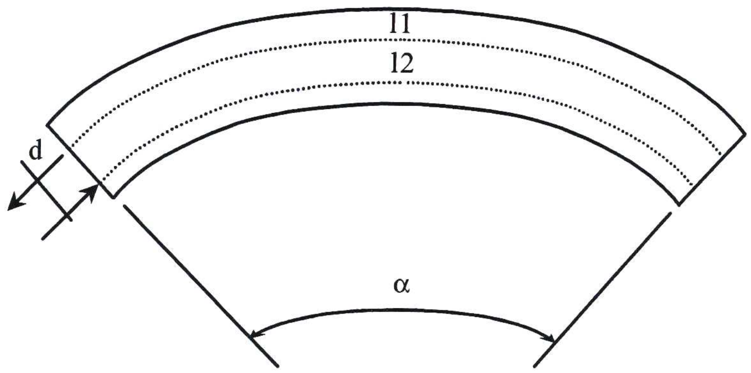
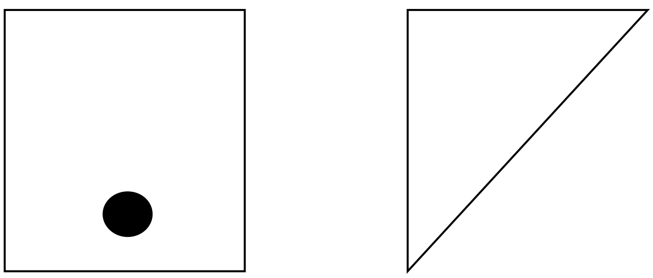


Figure 7.6: Application of the stresses

The beam does not have deformation from the empty force.



δ

Figure 7.7: Cable eccentricity and stresses in the concrete

In the transversal view, the force due to concrete is completely located upper the δ line passing by the tendon. In this case the link must take the total force and they will equal to:

$$A_s = \frac{p}{R \times \sigma_{sadm}}$$

In the case of beam-box where the bottom part has a curve with the average radius R.

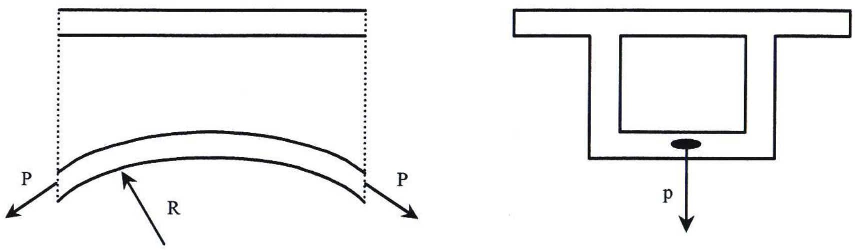


Figure 7.8: Transversal flexural deflection

The tendon in the bottom part of the beam is prestressed. The prestress force gives a distributed downward force p =P/R. The concrete cannot balance the force from the tendon, this means the bottom part of the beam is under a linear load p = P/R.

In this case shear force superimpose with the transversal bending over the bottom of the beam

Considering only one cable, the force by length unity in the bottom of the beam is:

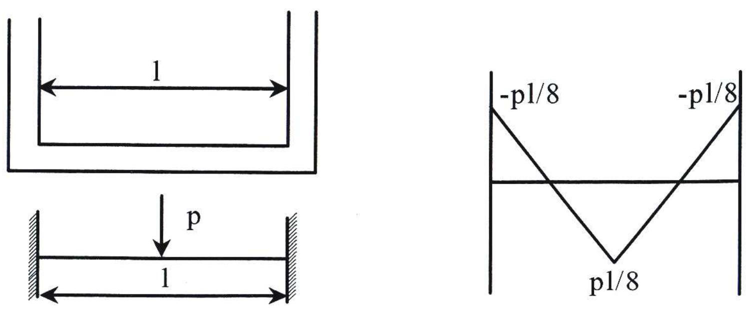


Figure 7.9: Moment in the bottom of the beam for one cable

Considering several cables, the force by length unity in the flange of the beam is:

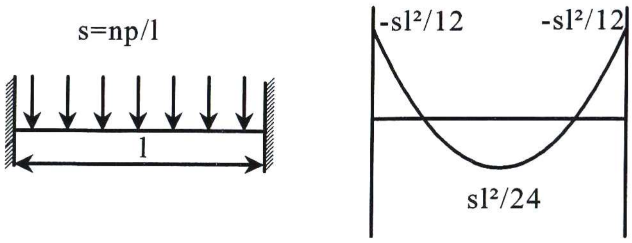


Figure 7.10: Moment in the bottom of the beam for several cables

In the two cases link must be put in the web to balance the reaction from the bottom flange.

Comment: The phenomenon can be analogue but in the opposite direction if the bottom flange will not have tendons and will be with only the upward concrete force.

<u>7.6- DEFLECTIONS:</u>

The deflection can be totally eliminated in the prestressed concrete members. This is realised by the use of a suitable arrangement of prestressing. The CAMBER of prestressed concrete members is the deflection when no applied load.

Concrete structure cannot predicted the deflections with a high accuracy because there are to many non-linear factors concerned, for example the concrete has not a linear stress-strain curve and the load-deflection characteristics of concrete beams are non-linear in general. The technique to know the exact deflection is to carry out tests on a model of the structure, using the same materials. The Eurocode 2 recommends for a structure where the sag of a member would be evident, the deflection under quasi-permanent load be limited to L/250, where L is the span of a bean or cantilever.

A basic method for finding the maximum deflection of concrete for cracked and uncracked members is using the shape of the bending moment diagram. The maximum deflection is given by the following:

$$y_{max} = KL^2 \times 1/r_b \qquad (4)$$

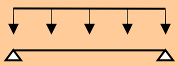
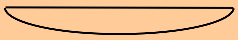
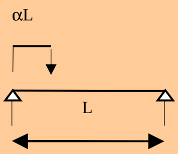
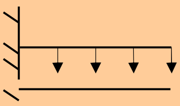
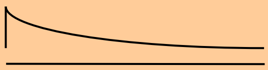
Where:

- L is the effective span
- $1/r_b$ is the curvature at midspan, or at the support for a cantilever ($1/r_b = 1/r_1$ for uncracked members and $1/r_1 = M_{qp}/EI$)
- Coefficient K can be obtained for simple loading with Table 3.12

Note:

- The deflection for complex loading must use other methods that follow Clause 7.4.3 checking the deflection by calculation of Part 1 to Eurocode 2.

Table 7.1: Coefficient K

Type of loading	Bending moment diagram	K
		0.104
		$\frac{3-4a^2}{48(1-\alpha)}$
		0.25
		0.125

7.7- THE SHEAR FORCES

The shear resistance of the prestressed concrete to Eurocode 2 is similar to the BS8110. At the ultimate limit state the shear resistance of prestressed concrete member is dependent on whether or not the section in the region of greatest shear force has cracked. The both situations are of the different actual mode of failure.

The Eurocode 2 considers only the cracked shear resistance of prestressed concrete members. This means there is the action of any longitudinal reinforcement crossing the shear crack, aggregate interlock across the two faces of the crack and the contribution of any vertical or inclined shear reinforcement crossing the crack.

It is habitually for beams with distributed loading to establish the shear resistance at the distance from the face of a support equal to *d*, the effective depth of the prestressing tendons.

The design of shear resistance in Eurocode 2

For a section of a prestressed concrete member with no shear reinforcement, the shear resistance V_{Rd1} is given by the following:

$$V_{Rd1} = [\ \tau_{Rd} k\ (1.2 + 40\ \vartheta_1) + 0.15\ \sigma_{cp}]\ b_w d \qquad (5)$$

Where:

- τ_{Rd} is the basic shear strength taken from table 3.13,
- k is equal to (1.6 – d), where d is not greater than 0.6 m,
- b_w is the minimum width of the section,
- ϑ_1 is the tension reinforcement ratio, with a maximum value of 0.02, defined as $(A_p + A_s) / b_{wd}$, where A_s is the area of untensioned reinforcement and A_p the area of any non-inclined prestressing tendons at the section considered,
- σ_{cp} is the axial stress arising from the prestressing force and any applied axial load.

Table 7.2: Basic concrete shear strength τ_{Rd} (N/mm²)

Concrete grade					
C25/30	*C30/37*	*C35/45*	*C40/50*	*C45/55*	*C50/60*
0.30	0.34	0.37	0.41	0.44	0.48

For a section of a prestressed concrete member with shear reinforcement there are two methods, but only the requirements of what is termed the 'standard' method:

The shear reinforcement has to be:

$$A_{sw}/s = (V_{sd} - V_{Rd1})/[0.78\ d\ f_{yk}(1 + \cot\alpha)\sin\alpha] \qquad (6)$$

Where:

- A_{sw} is the area of shear reinforcement over a length s
- f_{yk} is its characteristic strength
- α is the angle between the shear reinforcement and the longitudinal axis of the beam.
- V_{sd} is the ultimate shear force

This reinforcement may be a mix between the links and inclined bars, but at least 50% ought to be in the form of links.

The maximum shear resistance of a section, V_{Rd2} is given by the following:

$$V_{Rd2} = 0.3\ v\ f_{ck}\ b_w\ d\ (1 + \cot\alpha) \qquad (7)$$

Where:

- $v = 0.7 - f_{ck}/200$ $(v > 0.5)$
- If $\sigma_{cp} > 0.27\ f_{ck}$, the value of V_{Rd2} should be further reduced to $1.67\ (1 - 1.5\sigma_{cp}/f_{ck})\ V_{Rd2}$

If the applied shear force surpasses V_{Rd2} then the section size has to be increased.

If V_{sd} is less than V_{Rd1} then a minimum amount of shear reinforcement must be provided and given by the following:

$$A_{sw}/s = \vartheta_w b_w \sin\alpha \qquad (8)$$

Where:

- ϑ_w is the minimum shear reinforcement ratio, found from Table 3.14

Table 7.3: Minimum values for ϑ_w

Concrete grade	Steel type	
	Mild	High tensile
C25/30-C35/45	0.0024	0.0013
C40/50-C50/60	0.0030	0.0016

The maximum size of links should be 12mm for mild steel and the maximum spacing should not exceed the smaller of:

- 0.8 d, or 300 mm, if $V_{sd} < V_{Rd2}/5$
- 0.6 d, or 300 mm, if $V_{Rd2}/5 < V_{sd} < 2V_{Rd2}/3$
- 0.3 d, or 200 mm, if $V_{sd} > 2V_{Rd2}/3$

7.8- INEQUALITIES OF DESIGN OF THE MEMBERS

The beginning of this design process is to take a simply supported beam (see figure 3.8) carrying a uniform load. P_0 and e are respectively the prestress force and the eccentricity. The midspan stresses at the top and bottom fibres of the beam are at transfer, under the quasi-permanent and rare load combinations and showed by the equation given below. In the Eurocode 2, there is one moment at transfer but two moments at service, the rare load moment and the quasi-permanent load moment. Thus the inequalities work by five.

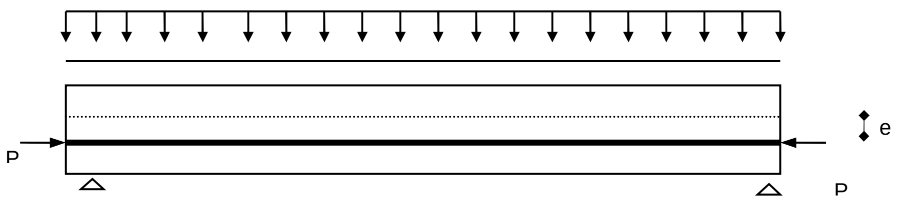


Figure 7.11: Rectangular beam

Transfer:

$$\sigma_t = \frac{\alpha P_0}{A_c} - \frac{\alpha P_0 e}{Z_t} + \frac{M_0}{Z_t} \qquad (9a)$$

$$\sigma_b = \frac{\alpha P_0}{A_c} + \frac{\alpha P_0 e}{Z_b} - \frac{M_0}{Z_b} \qquad (9b)$$

Quasi-permanent and rare load combinations:

$$\sigma_{tqp} = \frac{\beta P_0}{A_c} - \frac{\beta P_0 e}{Z_t} + \frac{M_{qp}}{Z_t} \qquad (9c)$$

$$\sigma_{tra} = \frac{\beta P_0}{A_c} - \frac{\beta P_0 e}{Z_t} + \frac{M_{ra}}{Z_t} \qquad (9d)$$

$$\sigma_b = \frac{\beta P_0}{A_c} + \frac{\beta P_0 e}{Z_b} - \frac{M_{ra}}{Z_b} \qquad (9e)$$

Where:

- Z_t, Z_b are the elastic section moduli for the top and the bottom fibres respectively
- A_c is the concrete cross-section
- α and β are the short- and long-term prestress loss factors respectively
- These equations assume that the transfer, quasi-permanent and rare load bending moments, M_o, M_{qp} and M_{ra} respectively, are sagging moments.

The maximum allowable compressive stresses in the concrete are f'_{max}, $(f_{max})_{qp}$ and $(f_{max})_{ra}$ at transfer, quasi-permanent and rare load respectively. The minimum stresses at transfer and rare load are f'_{min} and f_{min} respectively. If f_{min} is negative it ought indicate an permissible tensile stress. Then the equations can be written as inequalities:

$$\frac{\alpha P_0}{A_c} - \frac{\alpha P_0 e}{Z_t} + \frac{M_0}{Z_t} \geq f'_{min} \qquad (10a)$$

$$\frac{\alpha P_0}{A_c} + \frac{\alpha P_0 e}{Z_b} - \frac{M_0}{Z_b} \leq f'_{max} \qquad (10b)$$

$$\frac{\beta P_0}{A_c} - \frac{\beta P_0 e}{Z_t} + \frac{M_{qp}}{Z_t} \leq (f_{max})_{qp} \qquad (10c)$$

$$\frac{\beta P_0}{A_c} - \frac{\beta P_0 e}{Z_t} + \frac{M_{ra}}{Z_t} \leq (f_{max})_{ra} \qquad (10d)$$

$$\frac{\beta P_0}{A_c} + \frac{\beta P_0 e}{Z_b} - \frac{M_{ra}}{Z_b} \geq f_{min} \qquad (10e)$$

In the reality the inequalities (a) and (e) and the inequalities derived from them in the following sections, are the most critical cases, which is for minimum stress at transfer and under the quasi-permanent and rare load combinations. Yet for another member (i.e Double-T section with narrow ribs), the inequalities (b) can occasionally be critical. The criterion for determining the prestress force and eccentricity for some sections may be described in the following sections.

By combining Inequalities 9.2(a) and (c), and 9.2 (a) and (d), the expressions for Z_t might be derived:

$$Z_t \geq \frac{\alpha M_{qp} - \beta M_o}{\alpha (f_{max})_{qp} - \beta f'_{min}} \qquad (11a)$$

$$Z_t \geq \frac{\alpha M_{ra} - \beta M_o}{\alpha(f_{max})_{ra} - \beta f'_{min}} \quad (11b)$$

And by combining Inequalities 9.2(b) and (e), the expression for Z_b might be derived:

$$Z_b \geq \frac{\alpha M_{ra} - \beta M_o}{\beta f'_{max} - \alpha f_{min}} \quad (11c)$$

These two inequalities for the minimum values of Z_t, Z_b depend only on the difference between the maximum and minimum bending moments and allowable stresses, and not on their absolute values.

7.9- DESIGN OF PRESTRESS FORCE:

At this time the design process is to find the prestress force, based on a maximum eccentricity determined from the section properties.

After rearranging the inequalities:

$$P_0 \geq \frac{(Z_t f'_{min} - M_0)}{\alpha(Z_t / A_c - e)} \quad (12a)$$

$$P_0 \leq \frac{(Z_b f'_{max} + M_0)}{\alpha(Z_b / A_c + e)} \quad (12b)$$

$$P_0 \leq \frac{(Z_t (f_{max})_{qp} - M_{qp})}{\beta(Z_t / A_c - e)} \quad (12c)$$

$$P_0 \leq \frac{(Z_t (f_{max})_{ra} - M_{ra})}{\beta(Z_t / A_c - e)} \quad (12d)$$

$$P_0 \geq \frac{(Z_t f_{\min} - M_{ra})}{\beta(Z_b / A_c - e)} \qquad (12e)$$

These five inequalities have three upper and two lower bounds to the value of the prestress force. In general the minimum value is required since the cost of the prestressing steel is a significant proportion of the total cost of prestressed concrete structures.

7.10- MAGNEL DIAGRAM

The magnel diagram is designed with $1/P_o$ and not with P_o. The inequalities for prestress force are just rearranging. Nevertheless the value e may not be such a range, since the innermost of the bounds could overlap. In this case another value of e must be chosen and the limits for P_o found again, the process being repeated until a satisfactory combination of P_o and e is found. The inequalities can be given in the following form:

$$\frac{1}{P_0} \leq \frac{\alpha(Z_t / A_c - e)}{(Z_t f'_{\min} - M_0)} \qquad (13a)$$

$$\frac{1}{P_0} \geq \frac{\alpha(Z_b / A_c + e)}{(Z_b f'_{\max} + M_0)} \qquad (13b)$$

$$\frac{1}{P_0} \geq \frac{\beta(Z_t / A_c - e)}{(Z_t (f_{\max})_{qp} - M_{qp})} \qquad (13c)$$

$$\frac{1}{P_0} \geq \frac{\beta(Z_t / A_c - e)}{(Z_t (f_{\max})_{ra} - M_{ra})} \qquad (13d)$$

$$\frac{1}{P_0} \leq \frac{\beta(Z_b / A_c + e)}{(Z_b f_{\min} + M_{ra})} \qquad (13e)$$

Note:

- This inequalities are only valid if the denominators are positive, if it's the contrary (a), (c), (d) has been multiplied by a negative number and the sense of the inequality have to be reversed.

The magnel diagram has been introduced by a Belgian engineer, Magnel. The relations between $1/P_o$ and e are linear, and if plotted graphically, they offer very useful meanings to find the appropriate values of P_o and e.

7.11- CABLE ZONE

Following the choice of the value of the prestress force, the limit of the eccentricity e now may be found. At this step the term 'cable' is used to indicate the resultant of all the individual tendons. At the same time as the cable lies in the zone the stresses at the different loading stages will not exceed the allowed values, even though some tendons physically lie outside the cable zone.

$$e \le \frac{Z_t}{A_c} + \frac{1}{\alpha P_0}(M_0 - Z_t f'_{min}) \tag{14a}$$

$$e \le \frac{1}{\alpha P_0}(M_0 + Z_b f'_{max}) - \frac{Z_b}{A_c} \tag{14b}$$

$$e \ge \frac{Z_t}{A_c} + \frac{1}{\beta P_0}(M_{qp} - Z_t (f_{max})_{qp}) \tag{14c}$$

$$e \ge \frac{Z_t}{A_c} + \frac{1}{\beta P_0}(M_{ra} - Z_t (f_{max})_{ra}) \tag{14d}$$

$$e \ge \frac{1}{\beta P_0}(M_{ra} + Z_b f_{min}) - \frac{Z_b}{A_c} \tag{14e}$$

Figure 7.12: Flow chart for uncracked and cracked members

Uncracked	Cracked
Imposed load	Imposed load
Estimate self weight	Estimate self weight
Dead load	Dead load
Bending moment and shear force diagrams	Bending moment and shear force diagrams
Choose concrete grade and allowable stresses	- Choose concrete grade and allowable stresses
Determine min. Z_t and Z_b	Choose section
Choose section	Estimate prestress losses
Choose min. cover and determine max. e	Check Ultimate flexural strength
Estimate prestress losses	Choose SLS criterion
Determine prestress force limits	Choose no.and sizes of tendons
Choose no.and sizes of tendons	Choose cable profile
Determine cable zone and choose cable profile	Determine prestress losses
Determine prestress losses	Determine additional untensioned steel
Determine ultimate flexural strength	Check stresses and craking at SLS
Determine ultimate shear strength	Determine ultimate shear strength
End-block design or transmission length	End-block design or transmission length
Determine deflections	Determine deflections
Detailing	Detailing

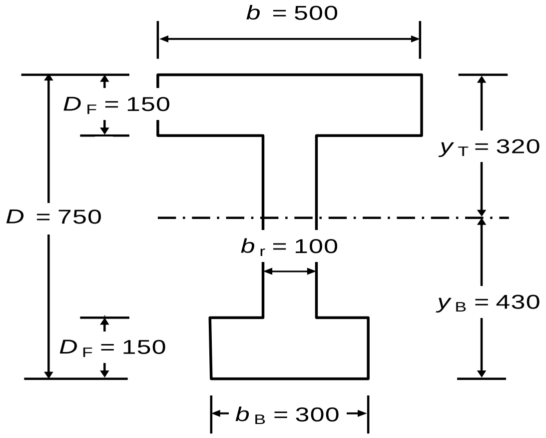
8- Design examples

This part presents the designs of three prestressed concrete member to the Eurocodes. The first example is a post-tensioned prestressed concrete I-beam. This member was designed to British code and showed by Doctor Ben Zhang during his courses. I followed the same process but I designed the members following the Eurocode 2. The second example is a pretensioned uncracked prestressed concrete rectangular beam. I designed this member following the design process of uncracked members to the Eurocode 2. These two first example come from my honours project, which was on prestressed concrete. Most of my calculations are of the losses of prestressed, the shear force and the deflection. The inequalities and equations used in this section are from part 4. The example 3 is an example of shear force design, for uncracked area

<u>Design of post-tensioned prestressed concrete I-beam simply supported over a span of 10.0 meter</u>

Using the design data provided, I will determine the tendons required and their eccentricity at mid-span(transfer at 7 days) and check the ultimate moment capacity of the section.

<u>Sectional properties of a prestressed concrete I-beam</u>



1.

Loading:

- dead load (excluding self-weight) = 10kN/m
- superimposed dead load = 6kN/m
- imposed load = 20kN/m

Material Properties:

- Concrete
  - f_{ck} = 50N/mm²
  - f'_{ck} = 36N/mm²
- Steel
  - f_{pk} = 1700N/mm²

Short term prestress loss at transfer = 12%
Long term prestress loss = 22%

The following cross-sectional properties for prestressed concrete beam are obtained from the section property tables for prestressed concrete.

- b_B/b = 0.6 b_r/b = 0.2
- D_F/D = 0.2 A_c/bD = 0.44
- y_T/D = 0.427 y_b/D = 0.573
- $I/bD3$ = 0.0535 Z_T/bD^2 = 0.0935
- Z_T/bD^2 = 0.1253

School of the Built Environment
Merchiston Campus
Napier University
10 Colinton Road
Edinburgh EH10 5DT

Project: Example 1
Job Title: Design post-tensioned beam
Design of prestressed concrete I-beam
Calculated by: Anthony Rochefort

No.	Calculations	Refs/Remarks
2.	**SOLUTION**	
2.1	**Design parameters for the prestressed concrete beams** For transfer at 7 days, the compressive strength of concrete for C50 is f'_{ck} = 36 N/mm² From part allowable stresses, the maximum allowable compressive concrete stress at transfer, f'_{max}, is $f'_{max} = 0.6\, f_{ck} = 0.6 \times 36 = 21.6$ N/mm² From part allowable stresses, the maximum allowable tensile concrete stress at transfer, $f'_{min,}$ is $f'_{min} = -3.2$ N/mm² For serviceability state, the compressive strength of concrete, f_{ck}, is $f_{ck} = 50$ N/mm² From part allowable stresses, the maximum allowable concrete stress under quasi-permanent load, $(f_{max})_{qp}$, is $(f_{max})_{qp} = 0.45\, f_{ck} = 0.45 \times 50 = 22.5$ N/mm² From part allowable stresses, the maximum allowable concrete stress under rare load, $(f_{max})_{ra}$, is $(f_{max})_{ra} = 0.6\, f_{ck} = 0.6 \times 50 = 30$ N/mm² From part allowable stresses, the maximum tensile stress allowed in The concreta at service is, $f_{min} = -\ 4.1$ N/mm² We take : $\alpha = 1 - 0.12 = 0.88$ $\beta = 1 - 0.22 = 0.78$ $\gamma = 24$ kN/m³ L = 10 m	
2.2	**Design loads for the prestressed concrete beams** UDL at transfer due to self-weight of the beam per metre width, w_o $w_o = \gamma\, A_c + w_d = 24 \times 165 \times 10^3 \times 10^{-6} + 10 = 13.96$ kN/m UDL at service due to self-weight and service load per metre width for quasi-permanent load (w_{qp}), frequent load (w_{fr}) and rare load (w_{ra}). $w_{qp} = w_o + 0.3\,(w_{imp} + w_{imp,d}) = 13.96 + 0.3\,(20+6) = 21.76$ kN/m $w_{fr} = w_{qp} + 0.3\,(w_{imp} + w_{imp,d}) = 21.76 + 0.3\,(20+6) = 29.56$ kN/m $w_{ra} = w_{fr} + 0.4\,(w_{imp} + w_{imp,d}) = 29.56 + 0.4\,(20+6) = 39.96$ kN/m Bending moment at transfer due to self-weight of the beam, M_0 $M_0 = w_o\, L^2 / 8 = 13.96 \times 10^2 / 8 = 174.5$ kNm Bending moment at service due to self-weight and service load, for quasi-permanent load (M_{qp}), frequent load (M_{fr}) and rare load (M_{ra}). $M_{qp} = wd\, L^2 / 8 = 21.76 \times 10^2 / 8 = 272.00$ kNm $M_{fr} = wd\, L^2 / 8 = 29.56 \times 10^2 / 8 = 369.50$ kNm $M_{ra} = wd\, L^2 / 8 = 39.96 \times 10^2 / 8 = 499.50$ kNm	

School of the Built Environment Merchiston Campus Napier University 10 Colinton Road	**Project: Example 1**	
	Job Title: Design post-tensioned beam	
	Design of prestressed concrete I-beam	
	Calculated by: Anthony Rochefort	

No.	Calculations	Refs/Remarks

2.3 **Elastic sectional moduli**

From Inequalities (11a) and (11b) and (11c), we can obtain the elastic section moduli

Required about the top and bottom fibres, Z_t and Z_b, as

$$Z_t \geq \frac{\alpha M_{qp} - \beta M_o}{\alpha (f_{max})_{qp} - \beta f'_{min}} = \frac{0.88 \times 272.00\times10^6 - 0.78 \times 174.5\times10^6}{0.88 \times 22.50 - 0.78 \times (-3.2)}$$

$$= \frac{124.31\times10^6}{22.296} = 5.57\times10^6 \text{ mm}^3 \ (< Z_t = 35.24\times 10^6 \text{ mm}^3 \text{ OK!})$$

$$Z_t \geq \frac{\alpha M_{ra} - \beta M_o}{\alpha (f_{max})_{ra} - \beta f'_{min}} = \frac{0.88 \times 499.50\times10^6 - 0.78 \times 174.5\times10^6}{0.88 \times 30 - 0.78 \times (-3.2)}$$

$$= \frac{303.45\times10^6}{28.896} = 10.50\times10^6 \text{ mm}^3 \ (< Z_t = 35.24\times 10^6 \text{ mm}^3 \text{ OK!})$$

$$Z_b \geq \frac{\alpha M_{ra} - \beta M_o}{\beta f'_{max} - \alpha f_{min}} = \frac{0.88 \times 499.50\times10^6 - 0.78 \times 174.5\times10^6}{0.78 \times 18 - 0.88 \times (-4.1)}$$

$$= \frac{303.45\times10^6}{17.65} = 17.19\times10^6 \text{ mm}^3 \ (< Z_b = 26.30\times 10^6 \text{ mm}^3 \text{ OK!})$$

2.4 **Determination of prestress force and eccentricity**

From Inequality (13a), we get

$$\frac{1}{P_o} \leq \frac{\alpha(Z_t/A_c - e)}{(Z_t f'_{min} - M_o)} - \frac{0.88 \times (35.24\times10^6 / 165000 - e)}{35.24\times10^6 \times (-3.2) - 174.5\times10^6}$$

$$= \frac{0.88 \times (213.58 - e)}{-287.26\times10^6} = \frac{-(0.3063e + 74.351)\times10^{-8}}{-1} \ \ 1/N$$

Note that the denominator is negative. Dividing both sides of an inequality by a negative number has effect of changing the sense of the inequality. Thus, the above inequality can be simplified as:

$$10^8 / P_o \geq 0.3063\, e - 74.351 \ \ 1/N \quad (i)$$

From Inequality (13b), we get

$$\frac{1}{P_o} \geq \frac{\alpha(Z_b/A_c + e)}{(Z_b f'_{max} + M_o)} = \frac{0.88 \times (26.30\times 10^6 / 165000 + e)}{26.30\times 10^6 \times 21.6 + 174.5\times10^6}$$

$$= \frac{0.88 \times (159.39 + e)}{742.58 \times10^6} = (0.1185e + 21.46)\times10^{-8} \ \ 1/N$$

$$10^8 / P_o \geq 0.1185\, e + 21.46 \ \ 1/N \quad (ii)$$

No.	**Calculations**	**Refs/Remarks**

From Inequality (13c), we get

$$\frac{1}{P_o} \geq \frac{\beta(Z_t/A_c - e)}{(Z_t(f_{max})_{qp} - M_{qp})} = \frac{0.78 \times (35.24\times10^6 / 165000 - e)}{35.24\times10^6 \times 22.5 - 272\times10^6}$$

$$= \frac{0.78 \times (213.58 - e)}{520.90 \times10^6} = (-0.1497e + 41.00)\times10^{-8} \ \ 1/N$$

$$10^8 / P_o \geq - 0.1497\, e + 41.00 \ \ 1/N \ \ (iii)$$

From Inequality (13d), we get

$$\frac{1}{P_o} \leq \frac{\beta(Z_t/A_c - e)}{(Z_t(f_{max})_{ra} - M_{ra})} = \frac{0.78 \times (35.24\times10^6 / 165000 - e)}{35.24\times10^6 \times 30 - 499.50\times10^6}$$

$$= \frac{0.78 \times (213.58 - e)}{557.70\times10^6} = (-0.1398e + 38.30)\times10^{-8} \ \ 1/N$$

$$10^8 / P_o \leq -0.1398\, e + 38.30 \ \ 1/N \ \ (iv)$$

From Inequality (13e), we get

$$\frac{1}{P_o} \leq \frac{\beta(Z_b/A_c - e)}{(Z_t\, f_{min} + M_{ra})} = \frac{0.78 \times (35.24\times10^6 / 165000 + e)}{35.24\times10^6 \times (-4.1) + 499.50\times10^6}$$

$$= \frac{0.78 \times (213.58 + e)}{355.016\times10^6} = (0.2197e + 60.16)\times10^{-8} \ \ 1/N$$

$$10^8 / P_o \leq 0.2197\, e + 60.16 \ \ 1/N \ \ (v)$$

Now we can put all four Inequalities together as :

$$10^8 / P_o \geq 0.3063\, e - 74.351 \ 1/N \quad (i)$$
$$10^8 / P_o \geq 0.1185\, e + 21.46 \ \ 1/N \quad (ii)$$
$$10^8 / P_o \geq - 0.1497\, e + 41.00 \ \ 1/N \ \ (iii)$$
$$10^8 / P_o \leq -0.1398\, e + 38.30 \ \ 1/N \ \ (iv)$$
$$10^8 / P_o \leq 0.2197\, e + 60.16 \ \ 1/N \ \ (v)$$

In this case, the distance from the neutral axis to the soffit is 430mm. If We logically take the distance from the centre of prestressing steel to the soffit as 80 mm, the maximum possible value of eccentricity for the permissible zone is

$$e = 430 - 80 = 350\text{mm}$$

The corresponding prestressing force will be obtained by

$$10^8 / P_o = 0.3063\, e - 74.351 = 0.3063 \times 350 - 74.351 = 32.854 \ 1/N$$

$$P_o \leq 10^8 / 32.854 = 3043.76 \text{ KN}$$

No.	**Calculations**	**Refs/Remarks**

$10^8 / P_o = 0.1185\ e + 21.46 = 0.1185 \times 350 + 21.46 = 62.94$ 1/N

$P_o \leq 10^8 / 62.94 = 1588.81$ KN

$10^8 / P_o = - 0.1497\ e + 41.00 = - 0.1497 \times 350 + 41 = - 11.395$ 1/N

$P_o \geq 10^8 / -11.395 = -8775.78$ KN

$10^8 / P_o = -0.1398\ e + 38.30 = -0.1398 \times 350 + 38.80 = -10.13$ 1/N

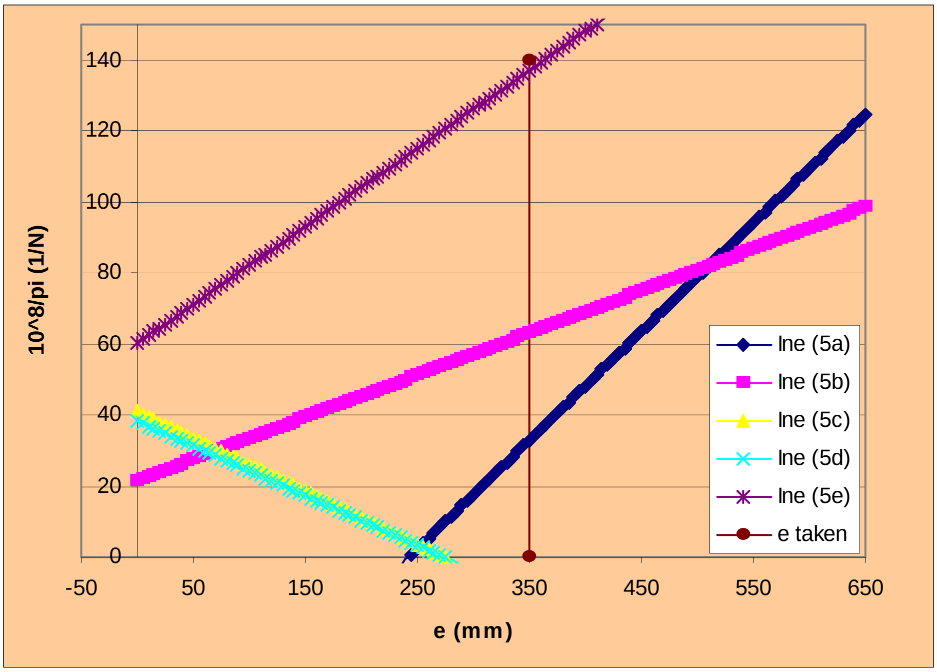
$P_o \geq 10^8 / -10.13 = -9407.34$ KN

$10^8 / P_o = 0.2197\ e + 60.16 = 0.2197 \times 350 + 60.16 = 137.055$ 1/N

$P_o \geq 10^8 / 137.055 = 729.634$ KN

$729.634 < P_o > 1588.81$

I chose to take P_o =1058.00 kN for assure the concrete stress at service



Magnel diagram

2.5 **Selection of prestressing steel**

I use the BS because there are not more explanation for this case inside Eurocode 2

BS8110:1997'At transfer the initial prestress should not exceed 70%'

$P_{req} = P_o / 0.7 = 1058/0.7 = 1511.42$kN

Here we assume 4No.7-wire super strands, so the required characteristic load per strand will be:

$P_{req} / 4 = 1511.42/4 = 377.85$kN

Try 4No.7-wire drawn strands of 18 nominal diameter with

$P_u = 380$kN/strand

With a nominal strength fpk = 1700N/mm² and a cross-sectional area of

$A_p = 223$mm²

Merchiston Campus Napier University 10 Colinton Road Edinburgh EH10 5DT	**Job Title: Design post-tensioned beam** **Design of prestressed concrete I-beam** **Calculated by: Anthony Rochefort**	

No.	Calculations	Refs/Remarks

The total prestress force P_u is calculated as

$$P_u = 380 \times 4 = 1520 \text{ kN}$$

with the total area of the prestressing steel A_{pu} is calculated as

$$A_{pu} = A_p \times 4 = 223 \times 4 = 892 \text{ mm}^2$$

The actual prestress force P_o is now becomes

$$P_o = 0.7\,P_u = 0.7 \times 1520 = 1064 \text{ kN}$$

2.6 **Concrete stresses at transfer**

From Inequality (9a), the stress at the top fibre f'_t is calculated as

$$f'_t = \frac{\alpha P_o}{A_c} - \frac{\alpha P_o e}{Z_t} + \frac{M_o}{Z_t}$$

$$= \frac{0.88 \times 1064 \times 10^3}{165000} - \frac{0.88 \times 1064 \times 10^3 \times 350}{35.24 \times 10^6} + \frac{174 \times 10^6}{35.24 \times 10^6}$$

$$= 5.67 - 9.30 + 4.94 = 1.31 \text{ N/mm}^2 > f'_{min} = -3.2 \text{ N/mm}^2 \text{ OK!}$$

From Inequality (9b), the stress at the bottom fibre f'_b is calculated as

$$f'_b = \frac{\alpha P_o}{A_c} + \frac{\alpha P_o e}{Z_b} - \frac{M_o}{Z_b}$$

$$= \frac{0.88 \times 1064 \times 10^3}{165000} + \frac{0.88 \times 1064 \times 10^3 \times 350}{26.30 \times 10^6} - \frac{174 \times 10^6}{26.30 \times 10^6}$$

$$= 5.67 + 12.46 - 6.62 = 11.51 \text{ N/mm}^2 < f'_{max} = 18 \text{ N/mm}^2 \text{ OK!}$$

2.7 **Concrete stresses at service**

From Inequality (9c), the stress at the top fibre f_t is calculated as

$$f'_{tqp} = \frac{\beta P_o}{A_c} - \frac{\beta P_o e}{Z_t} + \frac{M_{qp}}{Z_t}$$

$$= \frac{0.78 \times 1064 \times 10^3}{165000} - \frac{0.78 \times 1064 \times 10^3 \times 350}{35.24 \times 10^6} + \frac{272.00 \times 10^6}{35.24 \times 10^6}$$

$$= 5.03 - 8.24 + 7.72 = 4.51 \text{ N/mm}^2 < (f_{max})_{qp} = 22.5 \text{ N/mm}^2 \text{ OK!}$$

From Inequality (9d), the stress at the bottom f_b is calculated as

$$f'_{tra,} = \frac{\beta P_o}{A_c} - \frac{\beta P_o e}{Z_t} + \frac{M_{ra}}{Z_t}$$

$$= \frac{0.78 \times 1064 \times 10^3}{165000} - \frac{0.78 \times 1064 \times 10^3 \times 350}{35.24 \times 10^6} + \frac{499.5 \times 10^6}{35.24 \times 10^6}$$

$$= 5.03 - 8.24 + 14.17 = 10.96 \text{ N/mm}^2 < (f_{max})_{ra} = 30 \text{ N/mm}^2 \text{ OK!}$$

From Inequality (9e), the stress at the bottom f_b is calculated as

$$f'_{b,} = \frac{\beta P_o}{A_c} + \frac{\beta P_o e}{Z_b} - \frac{M_{ra}}{Z_b}$$

$$= \frac{0.78 \times 1024 \times 10^3}{165000} + \frac{0.78 \times 1024 \times 10^3 \times 350}{26.30 \times 10^6} - \frac{499.5 \times 10^6}{26.30 \times 10^6}$$

$$= 4.84 + 10.63 - 18.99 = -3.52 \text{ N/mm}^2 > f_{min} = -4.1 \text{ N/mm}^2 \text{ OK!}$$

School of the Built Environment Merchiston Campus Napier University 0 Colinton Road Edinburgh EH10 5DT	**Project: Example 1**	
	Job Title: Design post-tensioned beam	
	Design of prestressed concrete I-beam	
	Calculated by: Anthony Rochefort	

No.	**Calculations**	**Refs/Remarks**

2.8 **Bending moment resistance**

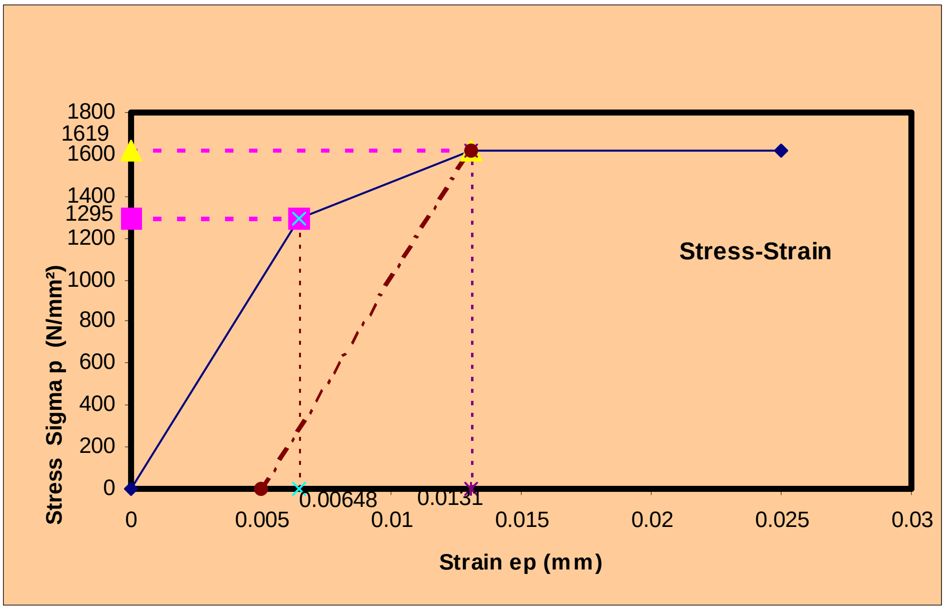
The initial stress in tendons is $f_{po} = 0.7\, f_{pu} = 0.7 \times 1700 = 1190$ N/mm²
The elastic modulus of steel is known as $E_p = 200$ kN/mm²

The stress-strain curve for the particular grade of steel used is shown in Figure with two typical points as follows

- transition point from linearly elastic to non-linear hardening
  $\sigma_{p,1} = 0.8\, f_{pu} / \gamma_m = ,0.8 \times 1700 / 1.05 = 1295$ N/mm²
  $\varepsilon_{p,1} = \sigma_{p,1} / E_p = 1295/(200\times10^3) = 0.00648$
- transition point from non-linear hardening to perfectly plastic
  $\sigma_{p,2} = f_{pu} / \gamma_m = 1700/ 1.05 = 1619$ N/mm²
  $\varepsilon_{p,2} = \sigma_{p,2} / E_p + 0.005 = 1619/(200\times10^3) + 0.005 = 0.01310$

The stress and strain distribution are shown in Figure 4.3. The strain in prestressing steel at the ultimate limit stat due to prestress only, ε_{pe}, is given by

$$\varepsilon_{pe} = 0.8\, f_{po} / E_p = 0.8 \times 1190/(200\times10^3) = 0.00476$$



Stress-strain diagram

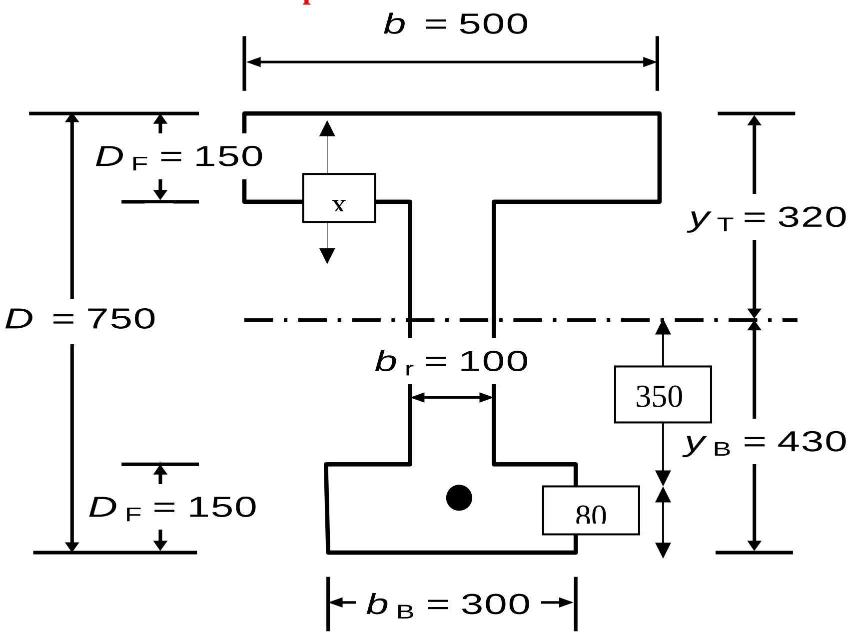
Here, ε_p is the strain in tendons due to flexure and is calculated from

$$\frac{0.0035}{x} = \frac{\varepsilon_p}{670-x}$$

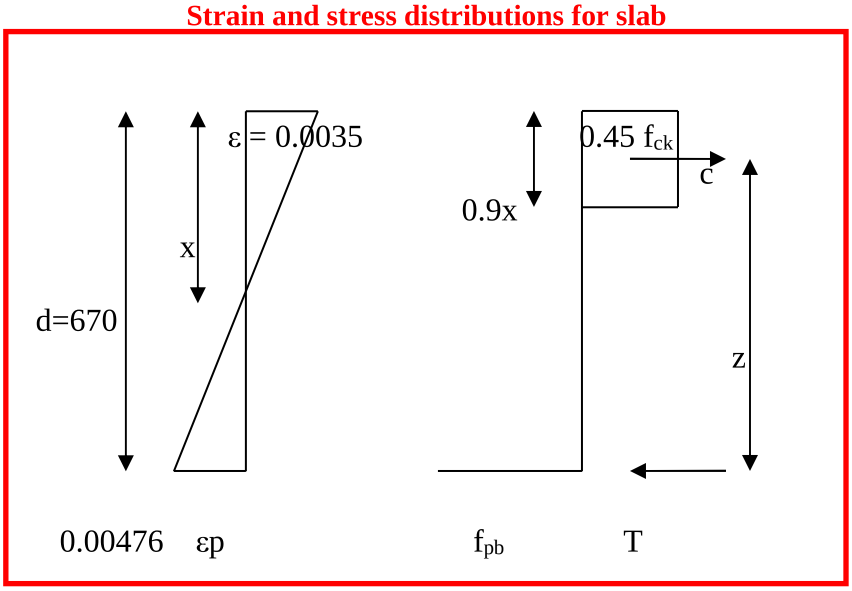
or $$\varepsilon_p = \frac{0.0035 \times (670-x)}{x}$$

School of the Built Environment Merchiston Campus Napier University 10 Colinton Road Edinburgh EH10 5DT	**Project: Example 1** **Job Title: Design post-tensioned beam** **Design of prestressed concrete I-beam** **Calculated by: Anthony Rochefort**

No.	Calculations	Refs/Remarks
	Compression zone for concrete	



Strain and stress distributions for slab



School of the Built Environment Merchiston Campus Napier University 10 Colinton Road	**Project: Example 1** **Job Title: Design post-tensioned beam** **Design of prestressed concrete I-beam** **Calculated by: Anthony Rochefort**

No.	Calculations	Refs/Remarks

The stress in the steel can be found from the stress-strain curve and the tensile and compressive forces in the steel and concrete, T and C, can then be determined according to the principles for the design of prestressed concrete sections. Here, we assume $0.9\,x > D_f$, so we can get

$$T = f_{pb} A_{ps} = \varepsilon_{pb} E_p A_{ps} = (\varepsilon_{pe} + \varepsilon_p) E_P A_{ps}$$

And $$C = [b D_f + b_r (0.9\,x - D_f)] (0.45 f_{ck})$$

We need to try different values of y or x to make sure $T = C$ first and then calculate the ultimate bending moment resistance

The following table 4.1 shows these forces for different values of y and x.

X (mm)	ε_p	ε_p	f_{pb} (N/mm2)	T (kN)	C (kN)
200	0.00823	0.01299	1613.6	1799.2	1755.0
205	0.00794	0.01270	1599.7	1783.6	1765.1
209	0.00772	0.01248	1589.0	1771.7	1773.2

when x =209 mm

$$\varepsilon_p = (670 - 209) \times 0.0035/209 = 0.00772$$
$$\varepsilon_{pb} = 0.00476 + 0.00772 = 0.01248$$
$$f_{pb} = 1295 + (1619-1295)\times(0.01248-0.00648)/(0.01310-0.00648)$$
$$= 1589.0 \text{ N/mm}^2$$
$$T = f_{pb} \times A_p = 1589 * 0.001115 = 1771.7 \text{ kN}$$
$$C = [b D_f + b_r (0.9\,x - D_f)] (0.45 f_{ck})$$
$$= [500 \times 150 + 100 \text{ x}(0.9 \text{ x } 209-150)] \text{ x } (0.45 \text{ x } 50) \text{ x}10^{-3}$$
$$= 1773.2\text{kN}$$

The neutral axis depth x may thus be taken with sufficient accuracy to be 209mm. In this case, the prestressing steel has not yielded.

Ultimate moment of resistance, M_{ult} may be calculated as

$$M_{ult} = (b D_f) (0.45 f_{ck}) (d - D_f/2) + b_r (0.9\,x - D_f)] (0.45 f_{ck})$$
$$[d - D_f - (0.9\,x - D_f)/2]$$
$$= [(500\times150) \times(045\times50)] \times (670 - 150/2) \times10^{-6}$$
$$+[100 \times(0.9\times209 - 150) \times(0.45 \times 50)]$$
$$\times [670 - 150 - (0.9\times209 - 150/2] \times10^{-6}$$
$$= 1004.06 + 42.94 = 1047.0\text{kNm}$$

where d is the effective depth for prestressing steel and

$$d = 750 - 80 = 670\text{mm}$$

No.	Calculations	Refs/Remarks

2.9 **Maximum bending moment due to applied loads**

Ultimate applied inform load (UDL), W_{ult}, is calculated as

$$W_{ult} = \gamma_g w_g + \gamma_q w_q$$
$$= 1.35 \times (13.96 + 6) + 1.5 \times 20$$
$$= 56.95 \text{kN/m}$$

Maximum ultimate bending moment, M_{max}, can now be calculated as

$$M_{max} = W_{ult} L^2 / 8$$
$$= 56.95 \times 10^2 / 8$$
$$= 671.83 \text{kNm}$$

2.10 **Summary**

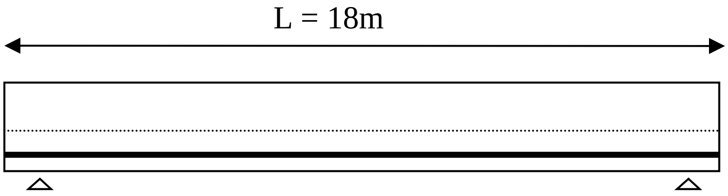
Here. The bending moment resistance, M_{ult}, is larger than the maximum bending moment due to applied loads, M_{max}, by 56%, so the present design is adequate to resist the ultimate bending moment.

No.	Calculations	Refs/Remarks

1.

<u>Design of pretensioned prestressed concrete of the rectangular beam simply supported over a span of 18.0 metres)</u>

Using the design data provided, I will determine the tendons required and their eccentricity at mid-span(transfer at 7 days) and determine the losses, the cable zone, the shear force and the deflection.



Loading:

- quasi-permanent load = 3kN/m
- frequent load = 4kN/m
- imposed load = 5kN/m
-

Material Properties:

- Concrete
  - f_{ck} = 40N/mm²
  - f'_{ck} = 30N/mm²
  - E_c = 35kN/mm²
- Steel
  - f_{pk} = 1820N/mm²
  - links f_{yk} =250N/mm²
  - Reinforcement f_{yk} = 460N/mm²

Short term prestress loss at transfer = 8%
Long term prestress loss = 20%

Merchiston Campus Napier University 10 Colinton Road Edinburgh EH10 5DT	**Job Title: Design pretensioned beam** **Design of rectangular uncracked beam** **Calculated by: Anthony Rochefort** **Ben Zhang**	

No.	**Calculations**	**Refs/Remarks**

2. **<u>SOLUTION</u>**

2.1 **<u>Design parameters for the prestressed concrete beams</u>**

For transfer at 7 days, the compressive strength of concrete for C40 is

$f'_{ck} = 30\ \text{N/mm}^2$

From part allowable stresses, the maximum allowable compressive concrete stress at transfer, f'_{max}, is

$f'_{max} = 0.6\ f_{ck} = 0.6 \times 30 = 18\ \text{N/mm}^2$

From part allowable stresses, the maximum allowable tensile concrete stress at transfer, f'_{min}, is

$f'_{min} = -2.9\ \text{N/mm}^2$

For serviceability state, the compressive strength of concrete, f_{ck}, is

$f_{ck} = 40\ \text{N/mm}^2$

From part allowable stresses, the maximum allowable concrete stress under quasi-permanent load, $(f_{max})_{qp}$, is

$(f_{max})_{qp} = 0.45\ f_{ck} = 0.45 \times 40 = 18\ \text{N/mm}^2$

From part allowable stresses, the maximum allowable concrete stress under rare load, $(f_{max})_{ra}$, is

$(f_{max})_{ra} = 0.6\ f_{ck} = 0.6 \times 40 = 24\ \text{N/mm}^2$

From part allowable stresses, the maximum tensile stress allowed in The concreta at service is,

$f_{min} = -\ 3.5\ \text{N/mm}^2$

We take : $\alpha = 1 - 0.08 = 0.92$ $\beta = 1 - 0.20 = 0.80$

$\gamma = 24\ \text{kN/m}^3$ $L = 18\ \text{m}$

Make an allowance of 8kN/m for the beam self weight,

Design loads for the prestressed concrete beams

$w_{qp} = 8 + 3 = 11\text{kN/m}$

$w_{fr} = 11 + 4 = 15\text{kN/m}$

$w_{ra} = 8 + 5 = 13\text{kN/m}$

At midspan the bnding moment at transfer due to self-weight of the beam, M_0

$M_0 = w_o\ L^2 / 8 = 8 \times 18^2 / 8 = 324\ \text{kNm}$

Bending moment at service due to self-weight and service load, for quasi-permanent load (M_{qp}), frequent load (M_{fr}) and rare load (M_{ra}).

$M_{qp} = w\text{d}\ L^2 / 8 = 11 \times 18^2 / 8 = 445.50\ \text{kNm}$

$M_{fr} = w\text{d}\ L^2 / 8 = 15 \times 18^2 / 8 = 607.50\ \text{kNm}$

$M_{ra} = w\text{d}\ L^2 / 8 = 13 \times 18^2 / 8 = 526.50\ \text{kNm}$

<table>
<tr><td rowspan="5">School of the Built Environment
Merchiston Campus
Napier University
10 Colinton Road
Edinburgh EH10 5DT</td><td colspan="2">Project: Example 2</td><td></td></tr>
<tr><td colspan="2">Job Title: Design pretensioned beam</td><td></td></tr>
<tr><td colspan="3">Design of rectangular uncracked beam</td></tr>
<tr><td colspan="2">Calculated by: Anthony Rochefort</td><td></td></tr>
<tr><td colspan="2"></td><td></td></tr>
<tr><td>No.</td><td colspan="2">Calculations</td><td>Refs/Remarks</td></tr>
</table>

2.3 **Elastic sectional moduli**

From Inequalities (11a) and (11b) and (11c), we can obtain the elastic section moduli

Required about the top and bottom fibres, Z_t and Z_b, as

$$Z_t \geq \frac{\alpha M_{qp} - \beta M_o}{\alpha (f_{max})_{qp} - \beta f'_{min}} = \frac{0.92 \times 445.50\times10^6 - 0.80 \times 324\times10^6}{0.92 \times 18 - 0.80 \times (-2.9)}$$

$$= \frac{150.66\times10^6}{18.88} = 7.98\times10^6 \text{ mm}^3$$

$$Z_t \geq \frac{\alpha M_{ra} - \beta M_o}{\alpha (f_{max})_{ra} - \beta f'_{min}} = \frac{0.92 \times 526.50\times10^6 - 0.80 \times 324\times10^6}{0.92 \times 24 - 0.80 \times (-2.9)}$$

$$= \frac{225.18\times10^6}{24.4} = 9.22\times10^6 \text{ mm}^3$$

$$Z_b \geq \frac{\alpha M_{ra} - \beta M_o}{\beta f'_{max} - \alpha f_{min}} = \frac{0.92 \times 526.50\times10^6 - 0.80 \times 324\times10^6}{0.80 \times 18 - 0.92 \times (-3.5)}$$

$$= \frac{225.18\times10^6}{17.62} = 12.77\times10^6 \text{ mm}^3$$

For an 840 mm deep by 380 mm wide section, $Z_t = Z_b = 44.69 \times10^6$ mm^3

$A_c = 319200$mm²

If the maximum eccentricity of the tendons at midspan is 70mm above the soffit, find the minimum value of prestress force required.

e_{max}= 840/2 - 70 = 350mm

w_o = 7.6 kN/m	M_0 = 307.8kN/m
w_{qp} = 10.6kN/m	M_{qp} = 429.3kN/m
w_{fr} = 14.6kN/m	M_{fr} = 591.3kN/m
w_{ra} = 12.6kN/m	M_{ra} = 510.3kN/m

2.4 **Determination of prestress force and eccentricity**

From Inequality (13a), we get

$$\frac{2}{P_o} \leq \frac{\alpha(Z_t/A_c - e)}{(Z_t f'_{min} - M_o)} = \frac{0.92 \times (44.69\times10^6 / 319200 - e)}{44.69\times10^6 \times (-2.9) - 307.8\times10^6}$$

$$= \frac{0.92 \times (140 - e)}{-437.40\times10^6} = \frac{-(0.2103e + 32.007)\times10^{-8}}{-1} \text{ 1/N}$$

Note that the denominator is negative. Dividing both sides of an inequality by a negative number has effect of changing the sense of the inequality. Thus, the above inequality can be simplified as:

$10^8 / P_o \geq 0.2103\, e - 32.007$ 1/N (i)

School of the Built Environment Merchiston Campus Napier University 10 Colinton Road Edinburgh EH10 5DT	**Project: Example 2**	
	Job Title: Design pretensioned beam	
	Design of rectangular uncracked beam	
	Calculated by: Anthony Rochefort	

No.	Calculations	Refs/Remarks
	From Inequality (13b), we get	

$$\frac{1}{P_o} \geq \frac{\alpha(Z_b/A_c + e)}{(Z_b f'_{max} + M_o)} = \frac{0.92 \times (44.69\times 10^6 \ / \ 319200 \ + e)}{44.69\times 10^6 \times 18 + 307.8\times 10^6}$$

$$= \frac{0.92 \times (140 + e)}{1112.2 \times 10^6} = (0.0827e + 12.59)\times 10^{-8} \ \ 1/N$$

$$10^8 / P_o \geq 0.0827e + 12.59 \ 1/N \ \ (ii)$$

From Inequality (13c), we get

$$\frac{1}{P_o} \geq \frac{\beta(Z_t/A_c - e)}{(Z_t(f_{max})_{qp} - M_{qp})} = \frac{0.80 \times (44.69\times 10^6 \ / \ 319200 - e)}{44.69\times 10^6 \times 18 - 429.3\times 10^6}$$

$$= \frac{0.80\times (140 - e)}{375.1 \times 10^6} = (-0.2133e + 37.23)\times 10^{-8} \ \ 1/N$$

$$10^8 / P_o \geq -0.2133e + 37.23 \ 1/N \ \ (iii)$$

From Inequality (13d), we get

$$\frac{1}{P_o} \leq \frac{\beta(Z_t/A_c - e)}{(Z_t(f_{max})_{ra} - M_{ra})} = \frac{0.80 \times (44.69\times 10^6 \ / \ 319200 - e)}{44.69\times 10^6 \times 24 - 510.3\times 10^6}$$

$$= \frac{0.80 \times (140 - e\ )}{562.26\times 10^6} = (-0.1423e + 24.90)\times 10^{-8} \ \ 1/N$$

$$10^8 / P_o \leq -0.1423\, e + 24.90 \ 1/N \ \ (iv)$$

From Inequality (13e), we get

$$\frac{1}{P_o} \leq \frac{\beta(Z_b/A_c - e)}{(Z_t\, f_{min} + M_{ra})} = \frac{0.80 \times (44.69\times 10^6 \ / \ 319200 + e)}{44.69\times 10^6 \times (-3.5) + 510.3\times 10^6}$$

$$= \frac{0.80 \times (140 + e\ )}{353.88\times 10^6} = (0.2261e + 39.56)\times 10^{-8} \ \ 1/N$$

$$10^8 / P_o \leq 0.2261\, e + 39.56 \ \ 1/N \ \ (v)$$

Now we can put all four Inequalities together as :

$$10^8 / P_o \geq \ 0.2103\, e - 32.007 \ 1/N \ \ (i)$$
$$10^8 / P_o \geq \ 0.0827e + 12.59 \ \ 1/N \ \ (ii)$$
$$10^8 / P_o \geq -\ 0.2133e + 37.23 \ \ 1/N \ \ (iii)$$
$$10^8 / P_o \leq \ -0.1423\, e + 24.90 \ 1/N \ \ (iv)$$
$$10^8 / P_o \leq \ 0.2261\, e + 39.56 \ \ 1/N \ \ (v)$$

Edinburgh EH10 5DT

No.	Calculations	Refs/Remarks

The corresponding prestressing force will be obtained by

$10^8 / P_o = 0.2103\,e - 32.007 = 0.2103 \times 350 - 32.007 = 41.60$ 1/N

$P_o \leq 10^8 / 41.60 = 2403.961$ KN

$10^8 / P_o = 0.0827e + 12.59 = 0.0827 \times 350 + 12.59 = 41.54$ 1/N

$P_o \leq 10^8 / 41.54 = 2407.32$ KN

$10^8 / P_o = -\,0.2133e + 37.23 = -\,0.2133 \times 350 + 37.23 = -\,37.43$ 1/N

$P_o \geq 10^8 / -37.43 = -2672.01$ KN

$10^8 / P_o = -0.1423\,e + 24.90 = -0.1423 \times 350 + 24.90 = -24.91$ 1/N

$P_o \geq 10^8 / -24.91 = -4014.45$ KN

$10^8 / P_o = 0.2261\,e + 39.56 = 0.2261 \times 350 + 39.56 = 118.64$ 1/N

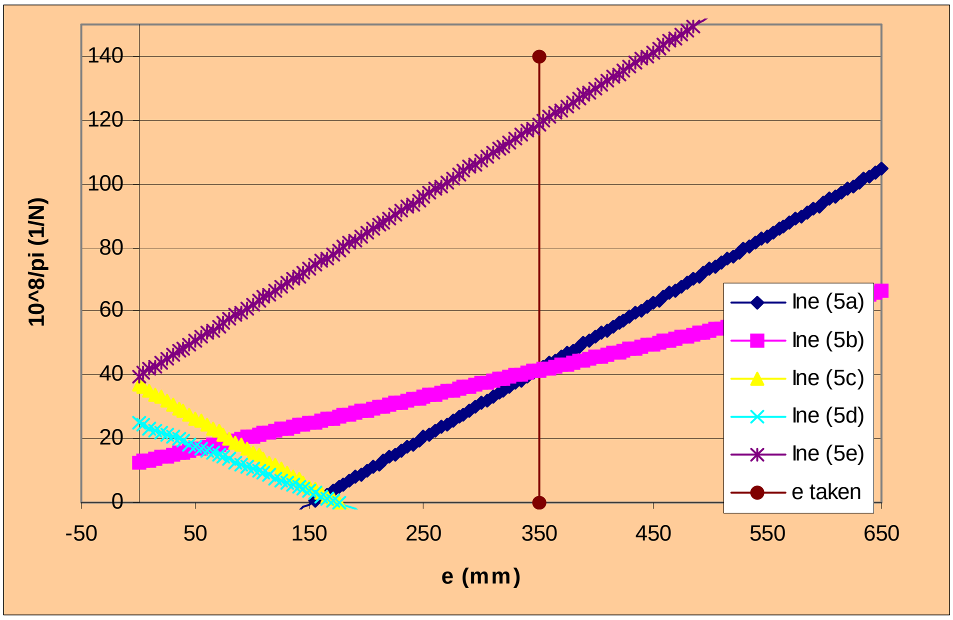
$P_o \geq 10^8 / 118.64 = 842.89$ KN

$$842.89 < P_o < 2403.961$$

If 7 15.7mm dia. Drawn strands are used, with $f_{pk} = 1820$ N/mm², the initial prestress force is given by

$$P_o = 7 \times 0.7 \times 165 \times 1820 \times 10^{-3} = 1471.47 \text{ kN} \quad \text{OK!!!}$$

Thus $A_p = 165 \times 7 = 1155$mm²



Magnel diagram

2.5

Estimate of Losses

2.5.1

Elastic shortening

$f_{po} = P_o / A_p = (1471.47 \times 10^3 / 1155) = 1274$N/mm²

$m = 200/E_{cm} = 200/32 = 6.25$

$I_c = bh^3/12 = (380 \times 840^3)/12 = 1.8769\ 10^{10}$ mm⁴

$r = (I_c/A_c)^{0.5} = (1.8769\ 10^{10} / 319200)^{0.5} = 242.5$mm

At midspan: (from equation(2))

$$f_{cg} = \frac{f_{po}}{\left[m + \dfrac{A_c}{A_p\,(1+e^2/r^2)}\right]} - \frac{M_o e}{I_c} = \frac{1274}{\left[6.25 + \dfrac{319200}{1155(1+350^2/242.5^2)}\right]} - \frac{307.8\ 10^6}{1.8769\ 10^{10}} = 13.27\text{N/mm}^2$$

School of the Built Environment
Merchiston Campus
Napier University

Project: Example 2
Job Title: Design pretensioned beam
Design of rectangular uncracked beam

10 Colinton Road Edinburgh EH10 5DT	**Calculated by: Anthony Rochefort**	

No.	**Calculations**	**Refs/Remarks**
	At support: $f_{cg} = \dfrac{f_{po}}{[m + \frac{A_c}{A_p}]} = \dfrac{1274}{[6.25 + \frac{319200}{1155}]} = 4.50\text{mm}^2$ $\Delta_{po1} = m \times f_{cgaverage} \times A_p = 6.25 \times (13.27+4.5)/2 \times 1155 10^{-3} = 64.14\text{N/mm}^2$	
2.5.2	**Long-term losses** Creep coefficient from table 4 Part 3: T_0 =365, relative humidity 50% $h_0 = (2A_c/u) = (2\times319200/((840+380)\times2) = 261$ thus creep coeff = 1.5 Shrinkage strain (from table 5 Part 3) $= 6\ 10^{-4}$ Low-relaxation steel: $\Delta\sigma_{pr}$= 38.22 N/mm² At midspan: $\sigma_{cg} = P_0 \times (1/A_C + e^{\wedge 2/Ic}) - M_{qp}\times e/I_c$ $= 1471.47\ 10^3 \times (1/319200 + 350^2/1.8769\ 10^{\wedge10}) - 445.5\ 10^6 \times 350 /1.8769\ 10^{10}$ $= 14.21 - 8.30 = 5.91\text{N/mm}^2$ $\sigma_{cpo} = 14.21\text{N/mm}^2$ Long-term losses $= \dfrac{\text{Shrinkage Strain}+\Delta\sigma_{pr} + m \times \text{Creep coeff} \times (\sigma_{cg+}\ \sigma_{cpo})}{(1+m \times (P_0 / A_c) \times(1+A_c\times e^2/Ic)\times (1+0.8 \times \text{Creep coeff})}$ $= \dfrac{6\ 10^{-4} \times 200\ 10^3 + 38.22 + 6.25 \times 1.5 \times (5.91 + 14.21)}{(1+6.25 \times (1471.47/319200)\times(1 + 319200\times350^2/1.8769\ 10^{\wedge10})\times(1+0.8 \times1.5)}$ $= \dfrac{346.845}{1.195} = 290.25\text{N/mm}^2$ At support: $\sigma_{cg} = \sigma_{cpo} = 14.21\ (7\text{-}3)/7 = 8.12\ \text{N/mm}^2$ Long term losses = $= \dfrac{\text{Shrinkage Strain}+\Delta\sigma_{pr} + m \times \text{Creep coeff} \times (\sigma_{cg+}\ \sigma_{cpo})}{(1+m \times (P_0 / A_c) \times(1+A_c\times e^2/Ic)\times (1+0.8 \times \text{Creep coeff})}$ $= \dfrac{6\ 10^{-4} \times 200\ 10^3 + 38.22 + 6.25 \times 1.5 \times (8.12+8.12)}{(1+6.25 \times (1471.47/319200)\times(1 + 319200\times350^2/1.8769\ 10^{\wedge10})\times(1+0.8 \times1.5)}$ $= \dfrac{310.47}{1.195} = 259.81\text{N/mm}^2$ Average long term losses = 275.03 N/mm² $\Delta_{po2} = 275.03 \times 1155 \times 10^{-3} = 317.66\text{kN}$ Mispan $\beta1 = P_o - (\Delta_{po1+}\ \Delta_{po2})/ P_o = (1155 - 64.14 - 275.03)/1155 = 0.71$	

School of the Built Environment Merchiston Campus	**Project: Example 2** **Job Title: Design pretensioned beam**	

No.	Calculations	Refs/Remarks

2.6 **Cable Zone**

From equation (14a)

$$e \le \frac{Z_t}{A_c} + \frac{1}{\alpha P_o}(M_o - Z_t f'_{min}) = \frac{44.69\times10^6}{319200} + \frac{1}{0.92\times1471.47\times10^3}(M_o - 44.69\times10^6\times(-2.9))$$

$$= 140.01 + 95.73 + 7.39\ 10^{-7} M_o$$

$$e \le 235.73 + 7.39\ 10^{-7} M_o$$

From equation (14b)

$$e \le \frac{1}{\alpha P_o}(M_o + Z_b f'_{max}) - \frac{Z_b}{A_c} = -\frac{44.69\times10^6}{319200} + \frac{1}{0.92\times1471.47\times10^3}(M_o + 44.69\times10^6\times(18))$$

$$= -140.01 + 594.21 + 7.39\ 10^{-7} M_o$$

$$e \le 454.21 + 7.39\ 10^{-7} M_o$$

From equation (14c)

$$e \ge \frac{Z_t}{A_c} + \frac{1}{\beta P_o}(M_{qp} - Z_t(f_{max})_{qp}) = \frac{44.69\times10^6}{319200} + \frac{1}{0.80\times1471.47\times10^3}(M_{qp} - 44.69\times10^6\times(18))$$

$$= 140.01 - 683.34 + 8.49\ 10^{-7} M_{qp}$$

$$e \ge -543.33 + 8.49\ 10^{-7} M_{qp}$$

From equation (14d)

$$e \ge \frac{Z_t}{A_c} + \frac{1}{\beta P_o}(M_{ra} - Z_t(f_{max})_{ra}) = \frac{44.69\times10^6}{319200} + \frac{1}{0.80\times1471.47\times10^3}(M_{ra} - 44.69\times10^6\times(24))$$

$$= 140.01 - 911.12 + 8.49\ 10^{-7} M_{ra}$$

$$e \ge -771.11 + 8.49\ 10^{-7} M_{ra}$$

From equation (14e)

$$e \ge \frac{1}{\beta P_o}(M_{ra} + Z_b f_{min}) - \frac{Z_b}{A_c} = -\frac{44.69\times10^6}{319200} + \frac{1}{0.80\times1471.47\times10^3}(M_{ra} - 44.69\times10^6\times(-3.5))$$

$$= -140.01 + 132.87 + 8.49\ 10^{-7} M_{ra}$$

$$e \ge -7.14 + 8.49\ 10^{-7} M_{ra}$$

We can put all the results together as

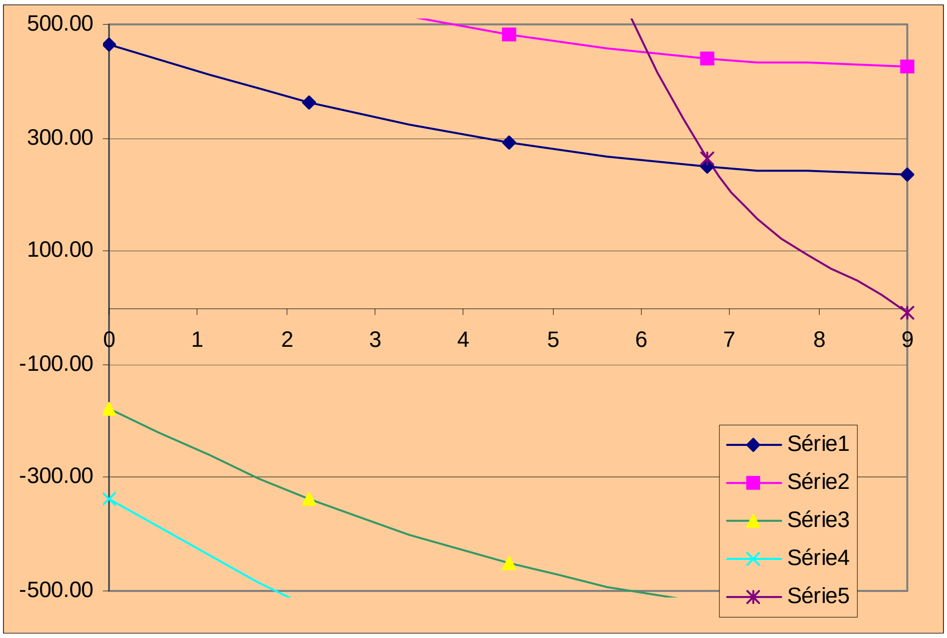
$$e \le 235.73 + 7.39\ 10^{-7} M_o$$
$$e \le 454.21 + 7.39\ 10^{-7} M_o$$
$$e \ge -543.33 + 8.49\ 10^{-7} M_{qp}$$
$$e \ge -771.11 + 8.49\ 10^{-7} M_{ra}$$
$$e \ge -7.14 + 8.49\ 10^{-7} M_{ra}$$

Table 4.2: Cable zone

Position	M_o (kNm)	M_{qp} (kNm)	M_{ra} (kNm)	Equation (6a)	(6b)	(6c)	(6d)	(6e)
0	0.00	0.00	0.00	235.73	425.21	-543.33	-771.11	-7.14
2.25	19.24	26.83	31.89	249.95	439.43	-520.55	-744.03	263.64
4.5	76.95	107.33	127.58	292.60	482.08	-452.21	-662.80	1075.97
6.75	173.14	241.48	287.04	363.68	553.16	-338.31	-527.41	2429.86
9	307.80	429.30	510.30	463.19	652.67	-178.85	-337.87	4325.31

Merchiston Campus Napier University 10 Colinton Road Edinburgh EH10 5DT	**Job Title: Design pretensioned beam**	
	Design of rectangular uncracked beam	
	Calculated by: Anthony Rochefort	

No.	**Calculations**	**Refs/Remarks**



Cable zone

2.7 **Concrete stresses at transfer**

From Inequality (1a), the stress at the top fibre f'_t is calculated as

$$f'_t = \frac{\alpha P_o}{A_c} - \frac{\alpha P_o e}{Z_t} + \frac{M_o}{Z_t}$$

$$= \frac{0.92 \times 1471\times10^3}{319200} - \frac{0.92 \times 1471\times10^3 \times 350}{44.69\times10^6} + \frac{307.8\times10^6}{44.69\times10^6}$$

$= 4.24 – 10.60 + 6.89 = 0.53$ N/mm² $> f'_{min} = -2.9$ N/mm² OK!

From Inequality (1b), the stress at the bottom fibre f'_b is calculated as

$$f'_b = \frac{\alpha P_o}{A_c} + \frac{\alpha P_o e}{Z_b} - \frac{M_o}{Z_b}$$

$$= \frac{0.92 \times 1471\times10^3}{319200} + \frac{0.92\times 1471\times10^3 \times 350}{44.69\times 10^6} - \frac{307.8\times10^6}{44.69\times 10^6}$$

$= 4.24 + 10.60 – 6.89 = 7.95$ N/mm² $< f'_{max} = 18$ N/mm² OK!

2.8 **Concrete stresses at service**

From Inequality (1c), the stress at the top fibre f_t is calculated as

$$f'_{tqp} = \frac{\beta P_o}{A_c} - \frac{\beta P_o e}{Z_t} + \frac{M_{qp}}{Z_t}$$

$$= \frac{0.80 \times 1471\times10^3}{319200} - \frac{0.80 \times 1471\times10^3 \times 350}{44.69\times10^6} + \frac{429.3\times10^6}{44.69\times10^6}$$

$= 3.69 – 9.22 + 9.61 = 4.08$ N/mm² $< (f_{max})_{qp} = 18$ N/mm² OK!

From Inequality (1d), the stress at the bottom f_b is calculated as

$$f'_{tra,} = \frac{\beta P_o}{A_c} - \frac{\beta P_o e}{Z_t} + \frac{M_{ra}}{Z_t}$$

<table>
<tr><td rowspan="5">School of the Built Environment
 Merchiston Campus
Napier University
10 Colinton Road
Edinburgh EH10 5DT</td><td colspan="2">Project: Example 2</td><td></td></tr>
<tr><td colspan="2">Job Title: Design pretensioned beam</td><td></td></tr>
<tr><td colspan="3">Design of rectangular uncracked beam</td></tr>
<tr><td colspan="2">Calculated by: Anthony Rochefort</td><td></td></tr>
<tr><td colspan="2"></td><td></td></tr>
<tr><td>No.</td><td colspan="2">Calculations</td><td>Refs/Remarks</td></tr>
</table>

$$= \frac{0.80 \times 1471\times10^3}{319200} - \frac{0.80 \times 1471\times10^3 \times 350}{44.69\times10^6} + \frac{510.3\times10^6}{44.69\times10^6}$$

$=3.69 - 9.22 + 11.42 = 5.90$ N/mm² < $(f_{max})_{ra} = 24$ N/mm² OK!

From Inequality (1e), the stress at the bottom f_b is calculated as

$$f'_{b,} = \frac{\beta P_o}{A_c} + \frac{\beta P_o e}{Z_b} - \frac{M_{ra}}{Z_b}$$

$$= \frac{0.80 \times 1471\times10^3}{319200} + \frac{0.80 \times 1471\times10^3 \times 350}{44.69\times10^6} - \frac{510.3\times10^6}{44.69\times10^6}$$

$= 3.69 + 9.22 - 11.42 = 1.48$ N/mm² > $f_{min} = -3.5$ N/mm² OK!

2.8 **Shear force:**

The effective depth of tendons is equal to 770mm

$w_{ult} = 1.35\times7.6 + 1.5\times5 = 17.76$kN/m ($M_{ult} = 719.28$kNm)

$V_{sd} = w_{ult} \times ((L/2) - 770\times10^{-3}) = 17.96 \times (9 - 0.77) = 209.92$kN

$M_{sd} = (w_{ult}\times770\times18\times0.77)/10^3 = 17.96 \times 770 \times18 \times0.77/10^3 = 191.67$kNm

Ro $= A_p/b_w d = 319200/770\times380 = 1.09\%$

$k = 1.6 - 0.77 = 0.83$ $\quad v = 0.7 - 40/200 = 0.5$

$\sigma_{cp} = P_0 \times \beta1 / A_c = 1471.47\ 10^3\times0.71 / 319200 = 3.15$N/mm²

Basic concrete shear strength $\tau_{Rd} = 0.41$N/mm² (from table13,part 3)

From equation (5)

$V_{Rd1} = [\ \tau_{Rd} k\ (1.2 + 40\ \vartheta_1) + 0.15\ \sigma_{cp}]\ b_w d$

$= [0.41\times1.6(1.2 + 40 \times0.00109) + 0.15 \times 3.15]\ (770\times380)/10^3$

$= 453.53$kN

From equation (7)

$V_{Rd2} = 0.3\ v f_{ck} b_w d\ (1 + \cot\alpha)$

$= 0.3 \times 0.5 \times 40 \times(380 \times 770)/10^3\ (1 + \cot 45)$

$= 3511$kN

From equation (6)

$A_{sw}/s = (V_{sd} - V_{Rd1})/[0.78\ d\ f_{yk}(1 + \cot\alpha)\sin\alpha]$

$= (209.92 - 453.53)10^3/(0.78 \times 770 \times 250 \times 1.41)$

$= -1.15$

Link diameter =15 mm

$s = 15^2\ \Pi / (2 \times (-1.15)) = -307$mm

$$Td = \frac{M_{sd}}{0.9d} + \frac{V_{sd}}{2} = \frac{191.67}{0.9\times0.77} + \frac{209.92}{2} = 381.54\text{kN}$$

$A_s = Td/0.87 \times f_{yk} = (381.54\times10^3)/(0.87\times460) = 953.37$ mm²

School of the Built Environment Merchiston Campus Napier University 10 Colinton Road Edinburgh EH10 5DT	**Project: Example 2** **Job Title: Design pretensioned beam** **Design of rectangular uncracked beam** **Calculated by: Anthony Rochefort**

No.	Calculations	Refs/Remarks

2.9 **Deflection:**

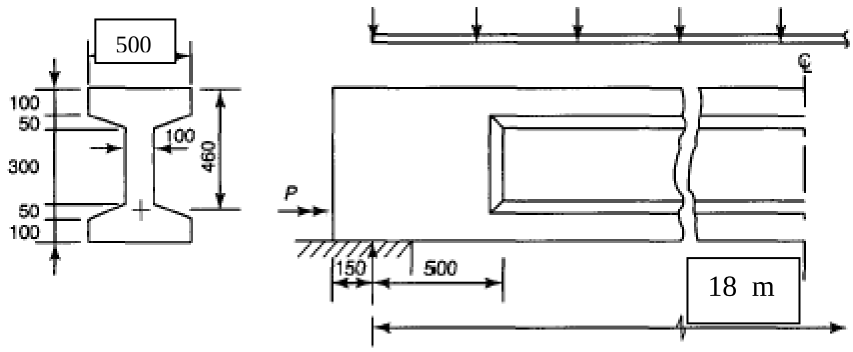
The basic method for finding the deflection of concrete members is using the shape of the bending moment:

$$y_{max} = KL^2 \times 1/r_b \quad \text{(from equation 3)}$$

$1/r_b$ will be $1/r_1$ because we assume a uncracked section.
M_{qp} = 429.3kN/m thus $1/r_1 = M_{qp}/EI = 429.3\times10^3/(35\times1.8769\ 10^{10}) = 6.54\times10^{-7}$
K (from table 12 of part 3) = 0.104

$$y_{max} = 0.104 \times 18^2\ 10^3 \times 6.54\times10^{-7} = 22.04\text{mm}$$

Example 3



Shear resistance to a prestressed beam

Nevertheless the poorer percentage of tensile reinforcement moderates the shear resistance. Two cases can be check. The first one is when a section has a reinforcement to check where there are bending crack. Second is where there are no bending crack but the main tensile stress in the section is high.

Taking the example of a pretensioned beam design to ultimate service state as show the figure above. The concrete grade is C30/40, the distributed design load is equal to 140 kN. The prestressing area is equal to 462 mm² with an initial prestressing force to 800 kN with 20 percent losses and 7mm wires stresses to 1850 Mpa.

Here I going to check when there are no bending crack but when the tensile stress in the section is high; because it is often the case where the tensile is high as the bending. This is known has stage two of the shear force check

- Check shear resistance of the uncracked web 0,5 m from the support.

Design shear force 0,5 m from the support

Ved = 65(9 - 0,5)/0,5 = 110.5 kN

Shear resistance of the web based on the principal tensile stress criterion

$$V_{rd,c} = Ibw / S(f^2{}_{ctd} + \alpha 1 \sigma_{cp} f_{ctd})^{0.5}$$

A = BD - bd = 500 x 600 - 400 x 350 = 160×10^3 mm^2

I = $BD^3/12$ - $bd^3/12$ = (500 x 600^3 - 400 x 350^3)/12 = 9085 × 10^6 mm^4

Z = Z/(D/2) = 9085E6/400 = 22,71× 10^6 mm^3

$$V_{rd,c} = 9085\times10^6 \times 100 / 16.38\times10^6 (2.73^2 + 0.611 + 5.93\times2.73)^{0.5} / 1000$$

$= 112.6$ kN > 110.5 kN, acceptable

where

$$S = B(D/2)\times(D/4) + (B - bW)\times(D - tf - 25)^2 / 2$$

$$S = 500\times300\times150 - 400\times(300 - 100 - 25)^2 / 2 = 16.38\times10^6 (approx)$$

lx= 650 mm

$$l_{pt} = \alpha 1 \alpha 2 \varphi \sigma_{pmo} / f_{bpt} = 1\times0.25\times7\times1850 / 3.65 = 886.98 mm$$

Where :

$$f_{btp} = \eta_{pt} \eta_t f_{ctd} = 2.7\times1\times1.35 = 3.65 Mpa$$

and

$$f_{ctd(t)} = \alpha_{ct} 0.7 f_{ctm(t)} / \gamma_c = 1.0\times0.7\times2.9 / 1.5 = 1.35 Mpa$$

$$l_{pt2} = 1.2 l_{pt} = 1.2\times886.98 = 1064.38 mm$$

$$\alpha = l_x / l_{pt2} = 650 / 1064.38 = 0.611$$

To finish to check the member, it must check the shear stress as a cracked (as a separately reinforced beam) by determine the section the closest to the support, where the cracking happens, in the bottom of the beam. It is possible at the end to calculate the compression reinforcement to resist shear force at the cracked section. It is known as stage one of the shear force check and the calculation above as stage two of the shear force check.

The principle of verification of torsion is calculated quite similarly for a prestressed beam. It checks the resistant moment is superior to the torsion moment. It is important to define the sign convention because the shear from the torsion in positive or negative is after addition to the shear force. The torsion reinforcement formula is given in the Eurocode.

General conclusion

To conclude, the Eurocodes give a good design approach for the prestress concrete and a good approach of the thermal action but the Eurocodes does not bring the two together. It is not easily comprehensible. One of the projects for an engineer is to link the Eurocodes to each other. I hope my dissertation gives a good start for the design of prestress concrete and thermal action. The creep and shrinkage are analysised in the Eurocodes, but the both phenomenon are not completely understood.

The key of my dissertation is to open the view of the engineer and explain to them, it is not because there is a code it is applicable and efficient for all kind of structures and researchers are trying to improve the knowledge on this point. It is also to make realise to the engineer that the influence of the temperature is most important.

Finally the prestress concrete is influenced by the temperature and deep research must be done to understand better and better the relationship between all the effects.
For the future, researchers have to improve the knowledge of the temperature effect to accurate the design and the Eurocodes.

References

1. Hurst M.K. (1998), Prestressed concrete design 2nd ed. E and FN Spon
2. Le Delliou (2003), Béton Précontraint aux Eurocodes. ENTPE PUL
3. Nawy E.G.(1999), Prestressed concrete A fundamental approach 3rd ed. Prentice Hakk
4. O'Brien E.J., Dixon A.S. (1995), Reinforced and Prestressed concrete design. Lonman Scientific and Technical
5. Zhang B.S., Bicanic N., Pearce C.J., Phillips, D.V. (2002), Relationship between brittleness and moisture loss of concrete exposed to high temperature. Cement and concrete., pp363-371
6. Zhang B.S., Bicanic N. (2001), Fracture energy of performance concrete at temperature up to 450°C. Proc. 4th Inter.Conf. on Fract. Mech. Of Conc. And Conc. Struc. Cachan, France, A.A. Balkema Publishers, pp 461-468
7. British Standard Institution (2004) BS EN 1992-1-1 General rules and rules for buildings, BSI.
8. British Standard Institution (2005) BS EN 1992-1-2 General rules-Structural fire design, BSI.
9. British Standard Institution (2004) BS EN 1991-1-1 Densities, self-weight and imposed loads, BSI.
10. British Standard Institution (2003) BS EN 1991-1-5 Thermal actions design, BSI.
11. British Standard Institution (2006) BS EN 1990 Basis of structure design, BSI.
12. Barr P.J., Stanton J.F., and Eberhard M.O., (2005) Effects of Temperature Variations on Precast, Prestressed Concrete Bridge Girders, JOURNAL OF BRIDGE ENGINEERING, Avril, pp.186-194
13. Lier H.P., Wittmann F.H. (1995) Coupled heat and mass transfer in concrete elements at elevated temperatures [Internet] Elsevier, institute for Building Materials. Swiss Federal Institute of Technology, CH-8093 Zirich, Switzerland. Available from: <http___www.emeraldinsight.com_Insight_ViewContentServlet_Filen

ame=_published_emeraldfulltextarticle_pdf_1100200403> [Accessed 6 March, 2008]

14. Powers T.C., (1968) The thermodynamics of volume change and creep, Materials an Structures: Research and Testing, Vol.1 No, 6, pp. 487-507
15. Feldman R.F., Sereda P.J., (1968) A model for hydrated Portland cement paste as deduced from sorption-length change and mechanical properties. Materials and Structures: Research and Testing, Vol.1, No.6, pp. 509-520
16. Rusch H., (1959) Physikalische Fragen der Betonprufung. (Physical problems in testing concrete) Zement-Klk-Gips, Vol.12, No,1, pp. 1-9
17. Jungwirth D., (1970) Determination of time varying creep and shrinkage deformation. An evaluation of field test and laboratory results. Sympostum of the Design of concrete structures for creep, shrinkage and temperature changes.
18. Acker P. and Hassan M., (1978) Prestress loss of thermal action during production of pretensioned concrete elements, Rep. LPC No, 78, Laboratoire Central des Ponts et Chaussees, Paris
19. AASHTO, (1989) AASHTO guide specification thermal effect in concrete brides superstructures, Washington DC
20. Concrete Q & A., (2007) "Estimating Evaporation Rates to Prevent Plastic Shrinkage Cracking", Concrete International
21. Menzies JB. and Gulvanessian H., (1998) Eurocode 1 The code for structural loading, Part 2. DETR, pp.1-8
22. Brown R.D. and al, (1973) The creep of structural concrete, Concrete Society Technical Paper No.101, pp.1-54
23. Nethercot D.A. and al, (2004) National strategy for implementation of the structural Eurocodes : Design Guidance, The Instituion of Structural Engineers, UK.
24. Concrete Society Technical report No,43, (2005) Post tensioned concrete floors Design handbook, Second Edition, Concrete Society, Camberley

25. Xu, Q and Burgoyne C., (2005) Effect of temperature and construction sequence on creep of concrete bridges. Proc.Inst. Civ. Engrs, Bridge Engineering
26. Burgoyne C., (2005) Analysis of continous Prestressed concrete beam. Proc.Inst. Civ. Engrs, Bridge Engineering
27. Freyssinet E., (1956). Birth of prestressing. Library translatiion 59, Cement and Concrete Association
28. Magnel G., (1951) Prestressed Concrete Ctatically Indeterminate Structures, Cement and Concrete Association, pp.77-86
29. Hambly E.C., (1991) Brigde deck behaiour (2 ed.). London
30. Emerson M., (1973) The calculation of the distribution of temperature in bridges, Technical report, Transport and Road Research Laboratory, LR561
31. Concrete society (2008), Movement, restraint and cracking in concrete structures, Technical report No.67, pp. 6-27
32. Yun L., Seong-Tae Y., Min-Su K. Jin-Keun K., (2006) Evaluation of a basic creep model with respect to autogenous shrinkage, Cement and concrete research 36, pp.1268-1278
33. Benboudjema F., Meftah F. and Torrenti J.M., (2005) Interaction between drying, shrinkage, creep and cracking phenomena in concrete, Engineering Structures 27,pp.239-250
34. Yuan Y. and Wan Z.L., (2002) Prediction of cracking within early-age concrete due thermal, drying and creep behaviour, Cement and concrete research 32,pp.1053-1059
35. Pane I. And Hansen W., (2008) Investigation on key properties controlling early-age stress development of blended cement concrete, Cement and concrete research, doi:10.1016/j.cemconres.2008.05.002
36. Yan Y. and Jianyon L.I., (2001) A study on creep and drying shrinkage of high performance concrete, Cement and concrete research 31, pp.1203-1206
37. Tia M., Liu Y. and Brown D., (2005) Modulus of elasticity, creep and shrinkage of concrete, Final report, U.F. Project No.49104504973-12

38. Acker P. and Ulm F., (2000) Creep and shrinkage of concrete: physical origins and practical measurements, Nuclear Engineering and Design 203, pp.143-158
39. Young J.F., (1980) The microstructure of hardened Portland Cement Paste, Creep and Shrinkage in concrete structures, pp. 3-22
40. Dougill J.W., (1980) Mechanics of concrete Systems; Current Approaches to Assessing Material Behaviour and Some Possible Extensions, Creep and Shrinkage in concrete structures, pp. 23-50
41. Cinlar E., (1980) Probabilistic Approach to Deformations of Concrete, Creep and Shrinkage in concrete structures, pp. 51-86
42. Pihlajavaara S.E., (1980) Estimation of Drying of Concrete at Different Relative Humidities and Temperatures, Creep and Shrinkage in concrete structures, pp. 87-107
43. Pomeroy C.D., (1980) Experimental Techniques and Results, Creep and Shrinkage in concrete structures, pp. 111-128
44. Wittmann F.H., (1980) Creep and Shrinkage Mechanisms, Creep and Shrinkage in concrete structures, pp. 129-162
45. Bazant Z.P., (1980) Mathematical Models for Creep and Shrinkage of Concrete, Creep and Shrinkage in concrete structures, pp. 163-257
46. Dilger W.H., (1980) Methods of structural Creep Analysis, Creep and Shrinkage in concrete structures, pp. 305-337
47. Samer A.Y. and Vistasp M.K., (2007)An approach to determine long-term behavior of concrete members prestressed with FRP tendons, Construction and buildings Materials 21,pp.1052-1060

Appendix

HISTORY OF PRESTRESSED CONCRETE:

The ideas of prestressed concrete is not new. The engineer wanted to find a solution for the low resistance in tension for the concrete long time ago.

In 1886, P. Jackson (USA) deposited the brevet for compressed the road, and in 1888, a German, W. Dohring deposited a brevet for prestressing the slab.

In 1907, Koenen and Lundt tested the cracking limitation under tension.

All experience was unsuccessful because the concrete was not of good quality, with a bad resistance.

In 1919, W.Wettstein used for the first time high strength steel.

In 1927, P. Forber invented a system for the sliding of the prestressing steel during the tension.

In October 1928, Eugène Freyssinet deposited the First Brevet for the prestressing. Inside the technology, the steel was able to resist the tension.

In 1929, Freyssinet requested his brevet for the firm Forclum who build electrics columns.

In 1933, Freyssinet reinforceed with success using prestressing the maritime station of the Havre.

In 1934, F. Dischinger and U. Finsterwalder gave the basic principle for external unbonded prestressing.

In 1937-1938, the first systems of external prestressing was used for the bridges.

At that moment the external prestressing was used for contoured the brevets of Freyssinet.

In 1940, E. Freyssinet found the process of wire and anchorage by concrete cone.

After the Second World War the prestressing systems rapidly extended with the works of Abeles in Great Britain, Magnel in Belgium, Leaonhardt in Germany, Lin in United States and Freyssinet and Guyon in France.

In 1948 and 1950, the news ideas appeared, including

- Bridges with partial prestressing (Abeles)
- Cables formed with tendons (Baur – Leonhardt)
- Using the U for build the prestressing bridges